

# DRHBc calculation result for odd-Z nuclei with $Z=61,63$ and $65$

The 7th workshop on nuclear mass table with  
DRHBc theory

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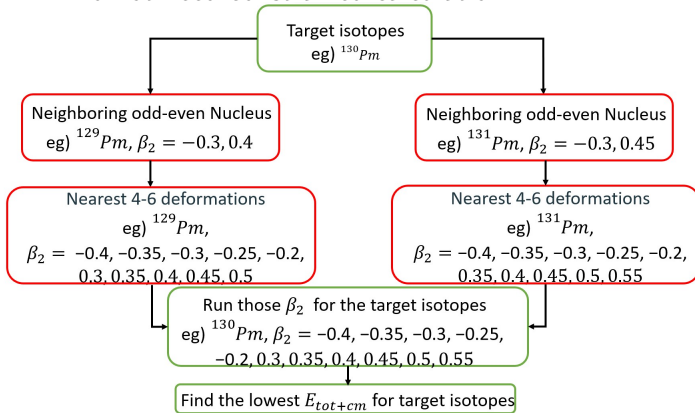
- 1 Methodology
- 2 Results
- 3 Summary and Future plan

# Odd-even nuclei ( $Z = 61, 63, 65$ )

- Constrained calculations for each isotope
  - Run each isotope with deformation from  $-0.40$  to  $0.55$ 
    - $\beta_2 = -0.40, -0.35, -0.30, \dots, 0.50, 0.55$
    - 20  $\beta_2$  per isotope
  - Figure out the  $\beta_2$  of local minimum  $E_{tot+cm}$
- Unconstrained calculations
  - Run 5 deformations with step sizes of  $0.05$  near  $\beta_2$  above
  - Unconstrained calculation with lowest  $E_{tot+cm}$   
 $\Rightarrow$  ground state

# Odd-odd nuclei ( $Z = 61, 63, 65$ )

- Follow the guideline from ManualforDRHBc\_202112.pdf
- With results from odd-even nuclei
  - Progress is simplified
  - Do not need constrained calculation





# Code version (odd- $Z$ nuclei)

- Odd-even isotopes
  - All odd-even isotopes  $61 \leq Z \leq 65$  are done (Code\_DRHBc\_202112)
- Odd-odd isotopes
  - All odd-odd isotopes  $61 \leq Z \leq 65$  are done
  - All unconstrained calculation: Code\_DRHBc\_202112
  - Constrained calculation for Pm( $Z = 61$ ):  
Code\_DRHBc\_202112
  - Constrained calculation for Eu( $Z = 63$ ) and Tb( $Z = 65$ ):  
Code\_DRHBc\_202401

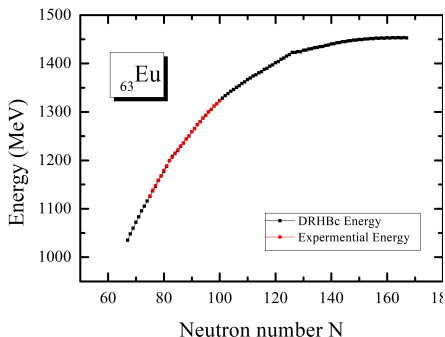
# Table summary

| N   | A   | Etot      | Ecm    | Etotcm    | E-rot  | Etotcmr1  | E_exp | beta n | beta p | beta t | radius | radius | radius | radius  | lam n   | lam p  | pairin  | pairin  | pairin  | pairin  | 52n | 51n     |
|-----|-----|-----------|--------|-----------|--------|-----------|-------|--------|--------|--------|--------|--------|--------|---------|---------|--------|---------|---------|---------|---------|-----|---------|
| 66  | 128 | -1014.833 | -6.624 | -1021.456 | -2.847 | -1024.303 | 0.385 | 0.404  | 0.395  | 4.854  | 4.895  | 4.874  | 4.960  | -12.678 | 0.319   | -3.590 | -5.307  | -8.897  |         |         |     |         |
| 67  | 129 | -1026.831 | -6.612 | -1033.444 | -2.460 | -1035.904 | 0.372 | 0.390  | 0.381  | 4.867  | 4.895  | 4.880  | 4.960  | -12.678 | -0.148  | 0.000  | -6.537  | -6.537  |         |         |     | -11.988 |
| 68  | 130 | -1039.414 | -6.519 | -1045.933 | -2.803 | -1048.736 | 0.366 | 0.383  | 0.374  | 4.884  | 4.900  | 4.891  | 4.965  | -12.041 | -0.585  | -3.847 | -7.143  | -10.990 | -24.477 | -12.490 |     |         |
| 69  | 131 | -1050.892 | -6.547 | -1057.439 | -2.584 | -1060.023 | 0.358 | 0.372  | 0.365  | 4.898  | 4.902  | 4.900  | 4.966  | -11.679 | -1.069  | 0.000  | -8.418  | -8.418  | -25.995 | -11.506 |     |         |
| 70  | 132 | -1062.782 | -6.547 | -1069.328 | -2.874 | -1072.203 | 0.354 | 0.366  | 0.360  | 4.914  | 4.905  | 4.910  | 4.970  | -11.196 | -1.508  | -0.265 | -9.706  | -9.371  | -25.995 | -11.890 |     |         |
| 71  | 133 | -1073.537 | -6.549 | -1080.085 | -2.685 | -1082.770 | 0.399 | 0.408  | 0.403  | 4.954  | 4.934  | 4.944  | 4.988  | -10.987 | -1.768  | 0.000  | -11.450 | -11.450 | -23.661 | -10.757 |     |         |
| 72  | 134 | -1085.219 | -6.536 | -1091.754 | -2.757 | -1094.512 | 0.438 | 0.443  | 0.440  | 4.993  | 4.962  | 4.979  | 5.026  | -10.682 | -1.991  | 0.000  | -6.721  | -6.721  | -22.426 | -11.669 |     |         |
| 73  | 135 | -1094.464 | -6.535 | -1100.999 | -2.494 | -1103.493 | 0.402 | 0.408  | 0.405  | 4.991  | 4.952  | 4.973  | 5.016  | -10.609 | -2.359  | 0.000  | -5.803  | -5.803  | -20.914 | -9.245  |     |         |
| 74  | 136 | -1105.665 | -6.322 | -1111.987 | -2.585 | -1114.572 | 0.233 | 0.255  | 0.243  | 4.936  | 4.889  | 4.915  | 4.954  | -10.661 | -2.405  | -5.710 | -11.034 | -16.744 | -20.233 | -10.988 |     |         |
| 75  | 137 | -1115.464 | -6.389 | -1121.854 | -2.313 | -1124.167 | 0.212 | 0.233  | 0.222  | 4.945  | 4.886  | 4.918  | 4.951  | -10.646 | -2.689  | -0.668 | -11.566 | -12.235 | -20.854 | -9.867  |     |         |
| 76  | 138 | -1126.563 | -6.236 | -1132.799 | -2.663 | -1135.463 | 0.183 | 0.208  | 0.195  | 4.956  | 4.886  | 4.925  | 4.951  | -10.398 | -2.862  | -6.205 | -12.746 | -18.951 | -20.812 | -10.946 |     |         |
| 77  | 139 | -1136.141 | -6.223 | -1142.364 | -2.357 | -1144.721 | 0.159 | 0.183  | 0.170  | 4.966  | 4.886  | 4.930  | 4.950  | -10.485 | -3.104  | -3.778 | -13.750 | -17.008 | -20.511 | -9.565  |     |         |
| 78  | 140 | -1147.267 | -6.149 | -1153.416 | -2.345 | -1155.761 | 0.127 | 0.149  | 0.137  | 4.976  | 4.884  | 4.935  | 4.949  | -10.475 | -3.390  | -5.534 | -14.967 | -20.501 | -20.617 | -11.052 |     |         |
| 79  | 141 | -1157.022 | -6.224 | -1163.246 | -1.571 | -1164.817 | 0.112 | 0.132  | 0.121  | 4.988  | 4.886  | 4.943  | 4.951  | -10.115 | -3.720  | 0.000  | -15.125 | -15.125 | -20.881 | -9.829  |     |         |
| 80  | 142 | -1168.359 | -6.104 | -1174.463 | 0.000  | -1174.663 | 0.000 | 0.000  | 0.000  | 4.994  | 4.880  | 4.945  | 4.945  | -10.978 | -3.991  | -6.117 | -15.684 | -21.801 | -21.047 | -11.218 |     |         |
| 81  | 143 | -1178.330 | -6.193 | -1184.723 | 0.000  | -1184.723 | 0.000 | 0.049  | 0.044  | 5.008  | 4.885  | 4.955  | 4.950  | -8.902  | -4.317  | 0.000  | -15.554 | -15.554 | -21.477 | -10.259 |     |         |
| 82  | 144 | -1190.090 | -6.206 | -1196.296 | 0.000  | -1196.296 | 0.000 | 0.000  | 0.000  | 5.020  | 4.887  | 4.963  | 4.952  | -7.849  | -4.594  | 0.000  | -15.690 | -15.690 | -21.832 | -11.573 |     |         |
| 83  | 145 | -1195.460 | -6.195 | -1201.656 | 0.000  | -1201.656 | 0.000 | -0.051 | -0.046 | 5.044  | 4.903  | 4.984  | 4.968  | -6.835  | -4.932  | 0.000  | -15.196 | -15.196 | -16.933 | -5.360  |     |         |
| 84  | 146 | -1202.532 | -6.028 | -1208.561 | 0.000  | -1208.561 | 0.000 | 0.000  | 0.000  | 5.064  | 4.911  | 5.000  | 4.975  | -6.197  | -5.209  | -8.238 | -15.522 | -23.760 | -12.265 | -6.905  |     |         |
| 85  | 147 | -1208.497 | -6.160 | -1214.657 | -1.130 | -1215.788 | 0.126 | 0.135  | 0.130  | 5.096  | 4.937  | 5.030  | 5.002  | -7.319  | -5.633  | 0.000  | -14.047 | -14.047 | -19.001 | -6.097  |     |         |
| 86  | 148 | -1216.059 | -6.060 | -1222.119 | -1.898 | -1224.017 | 0.152 | 0.162  | 0.156  | 5.120  | 4.951  | 5.050  | 5.015  | -7.109  | -5.955  | -4.223 | -13.631 | -17.854 | -15.598 | -7.462  |     |         |
| 87  | 149 | -1222.735 | -6.121 | -1228.856 | -1.557 | -1230.413 | 0.186 | 0.201  | 0.192  | 5.146  | 4.971  | 5.074  | 5.035  | -6.938  | -6.292  | 0.000  | -12.708 | -12.708 | -14.158 | -6.736  |     |         |
| 88  | 150 | -1230.371 | -6.068 | -1236.438 | -1.890 | -1238.328 | 0.215 | 0.232  | 0.222  | 5.172  | 4.987  | 5.096  | 5.051  | -7.047  | -6.593  | -2.449 | -11.266 | -13.716 | -14.319 | -7.583  |     |         |
| 89  | 151 | -1236.810 | -6.163 | -1242.972 | -1.463 | -1244.435 | 0.254 | 0.277  | 0.263  | 5.201  | 5.011  | 5.124  | 5.074  | -7.156  | -6.951  | 0.000  | -9.796  | -7.996  | -14.117 | -6.534  |     |         |
| 90  | 152 | -1244.488 | -6.125 | -1250.612 | -2.114 | -1252.726 | 0.301 | 0.322  | 0.309  | 5.237  | 5.039  | 5.157  | 5.102  | -7.225  | -7.195  | -4.591 | -4.791  | -9.382  | -14.174 | -7.640  |     |         |
| 91  | 153 | -1251.243 | -6.209 | -1257.452 | -1.874 | -1259.326 | 0.314 | 0.332  | 0.321  | 5.258  | 5.052  | 5.176  | 5.115  | -6.844  | -7.428  | 0.000  | -4.555  | -4.555  | -14.480 | -6.840  |     |         |
| 92  | 154 | -1258.855 | -6.208 | -1264.799 | -2.126 | -1266.919 | 0.339 | 0.353  | 0.345  | 5.286  | 5.070  | 5.200  | 5.133  | -6.636  | -7.666  | -0.007 | -4.574  | -4.581  | -14.181 | -7.341  |     |         |
| 93  | 155 | -1264.373 | -6.161 | -1270.534 | -2.051 | -1272.585 | 0.342 | 0.354  | 0.347  | 5.304  | 5.077  | 5.215  | 5.140  | -6.548  | -7.956  | 0.000  | -4.698  | -4.698  | -13.082 | -5.741  |     |         |
| 94  | 156 | -1271.027 | -6.048 | -1277.076 | -2.467 | -1279.542 | 0.347 | 0.358  | 0.351  | 5.324  | 5.089  | 5.232  | 5.152  | -6.126  | -8.382  | -5.851 | -4.453  | -10.304 | -12.283 | -6.542  |     |         |
| 95  | 157 | -1276.382 | -6.060 | -1282.442 | -2.371 | -1284.813 | 0.352 | 0.360  | 0.355  | 5.343  | 5.100  | 5.248  | 5.162  | -5.912  | -8.761  | -3.544 | -4.360  | -7.904  | -11.908 | -5.366  |     |         |
| 96  | 158 | -1283.008 | -5.988 | -1288.996 | -2.527 | -1291.522 | 0.356 | 0.364  | 0.359  | 5.362  | 5.109  | 5.264  | 5.172  | -5.934  | -9.090  | -6.529 | -4.467  | -10.996 | -11.920 | -6.554  |     |         |
| 97  | 159 | -1288.202 | -6.034 | -1294.236 | -2.112 | -1296.349 | 0.358 | 0.364  | 0.360  | 5.379  | 5.118  | 5.279  | 5.180  | -5.779  | -9.452  | -1.795 | -4.471  | -6.267  | -11.794 | -5.240  |     |         |
| 98  | 160 | -1294.581 | -5.970 | -1300.551 | -2.440 | -1302.991 | 0.359 | 0.372  | 0.370  | 5.401  | 5.131  | 5.298  | 5.193  | -5.696  | -9.765  | -4.929 | -4.518  | -9.446  | -11.595 | -6.315  |     |         |
| 99  | 161 | -1299.837 | -6.034 | -1305.871 | -2.055 | -1307.926 | 0.376 | 0.378  | 0.377  | 5.420  | 5.142  | 5.315  | 5.204  | -5.220  | -10.101 | 0.000  | -4.450  | -4.450  | -11.034 | -5.320  |     |         |
| 100 | 162 | -1305.612 | -5.994 | -1311.606 | -2.040 | -1313.646 | 0.379 | 0.380  | 0.380  | 5.441  | 5.151  | 5.332  | 5.213  | -5.134  | -10.368 | -0.002 | -4.589  | -4.591  | -11.655 | -5.735  |     |         |
| 101 | 163 | -1310.038 | -5.997 | -1316.036 | -2.260 | -1318.296 | 0.385 | 0.385  | 0.385  | 5.459  | 5.162  | 5.348  | 5.224  | -5.100  | -10.771 | 0.000  | -4.445  | -4.445  | -10.165 | -4.430  |     |         |
| 102 | 164 | -1314.903 | -5.880 | -1320.783 | -2.566 | -1323.349 | 0.383 | 0.382  | 0.383  | 5.478  | 5.170  | 5.364  | 5.231  | -4.444  | -11.005 | -5.130 | -4.403  | -9.932  | -9.177  | -4.747  |     |         |
| 103 | 165 | -1318.752 | -5.939 | -1324.691 | -2.306 | -1326.997 | 0.455 | 0.434  | 0.447  | 5.536  | 5.210  | 5.416  | 5.271  | -4.405  | -11.027 | 0.000  | -3.567  | -3.567  | -8.655  | -3.908  |     |         |
| 104 | 166 | -1323.551 | -5.873 | -1329.424 | -2.536 | -1331.961 | 0.450 | 0.430  | 0.442  | 5.550  | 5.216  | 5.427  | 5.277  | -4.251  | -11.362 | -3.488 | -3.729  | -7.218  | -8.641  | -4.734  |     |         |
| 105 | 167 | -1327.178 | -5.888 | -1333.066 | -2.290 | -1335.356 | 0.449 | 0.429  | 0.442  | 5.567  | 5.222  | 5.441  | 5.283  | -4.083  | -11.587 | 0.000  | -3.998  | -3.999  | -8.375  | -3.642  |     |         |

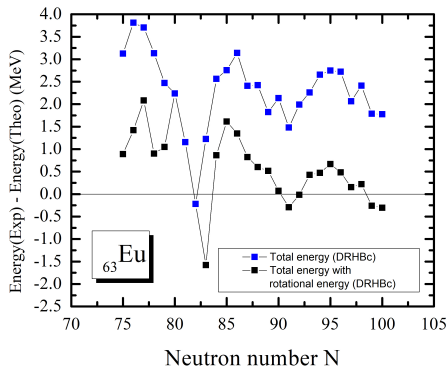
Results are being double-checked

# Ground state energy: Eu ( $Z = 63$ )

Ground state energy for Eu  
( $Z = 63$ )



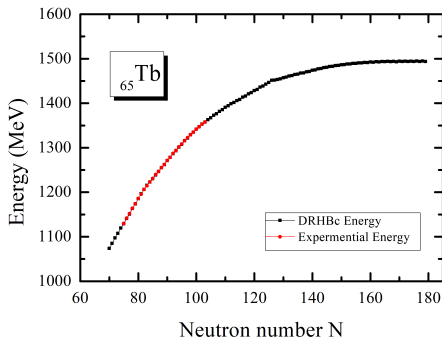
Difference between the  
experimental energy and DRHBc  
calculations from  ${}^{137}\text{Eu}$  to  ${}^{165}\text{Eu}$



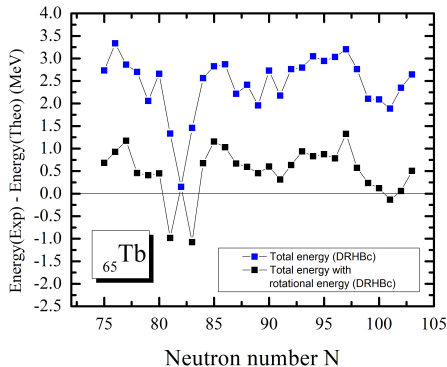
With rotational energy  $\Rightarrow$  better prediction (Except  ${}^{146}\text{Eu}$ )

# Ground state energy: Tb ( $Z = 65$ )

Ground state energy for Tb  
( $Z = 65$ )



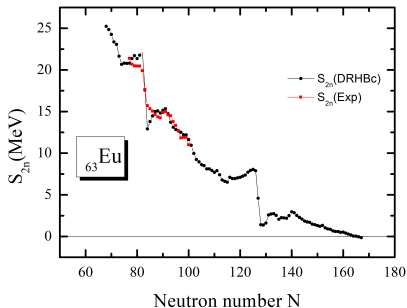
Difference between the  
experimental energy and DRHBc  
calculations from  $^{141}\text{Tb}$  to  $^{167}\text{Tb}$



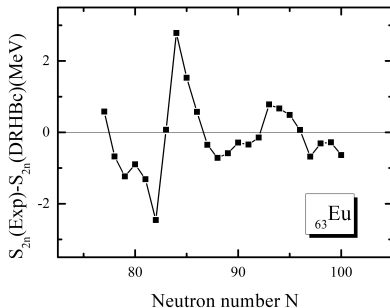
With rotational energy  $\Rightarrow$  better prediction

# 2-neutron Separation Energy: Eu ( $Z = 63$ )

2-neutron separation Energy for Eu ( $Z = 63$ )



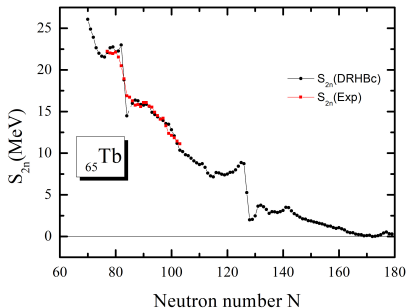
Difference between experimental  $S_{2n}$  and DRHBc  $S_{2n}$  from  $^{139}\text{Eu}$  to  $^{165}\text{Eu}$



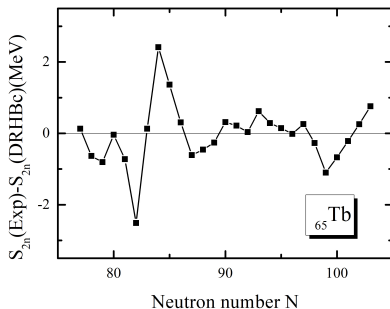
Agree with shell closure, but description needs improvement

# 2-neutron Separation Energy: Tb ( $Z = 65$ )

2-neutron separation Energy for Tb ( $Z = 65$ )



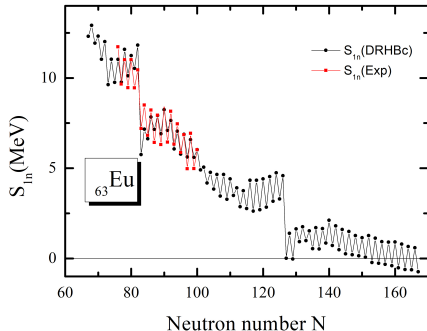
Difference between experimental  $S_{2n}$  and DRHBc  $S_{2n}$  from  ${}^{143}\text{Tb}$  to  ${}^{167}\text{Tb}$



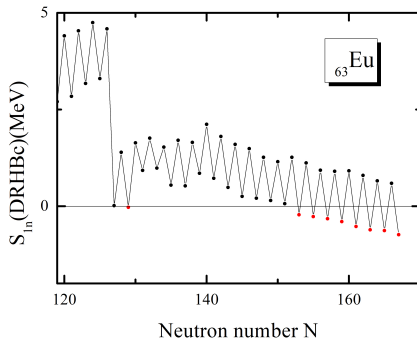
Same phenomenon can be concluded in Tb ( $Z = 65$ )

# 1-neutron Separation Energy: Eu ( $Z = 63$ )

Single neutron separation Energy for Eu ( $Z = 63$ )



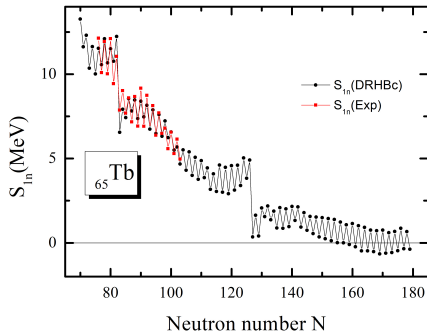
Single neutron separation Energy for Eu ( $Z = 63$ ) from  ${}^{183}\text{Eu}$  to  ${}^{230}\text{Eu}$



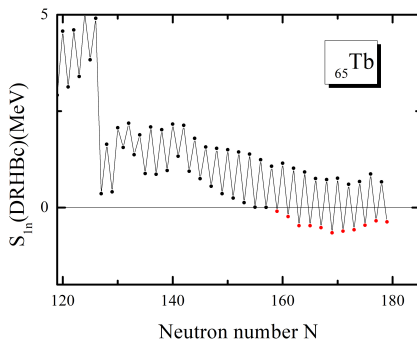
1 unbounded nuclei exist in  ${}^{192}\text{Eu}$

# 1-neutron Separation Energy: Tb ( $Z = 65$ )

Single neutron separation Energy for Tb ( $Z = 65$ )



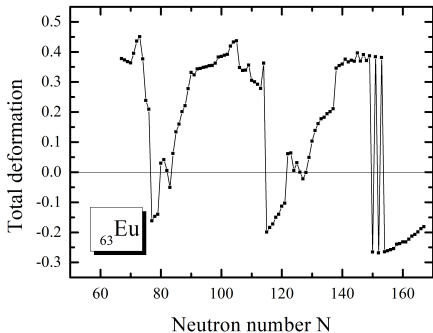
Single neutron separation Energy for Tb ( $Z = 65$ ) from  ${}^{185}\text{Tb}$  to  ${}^{244}\text{Tb}$



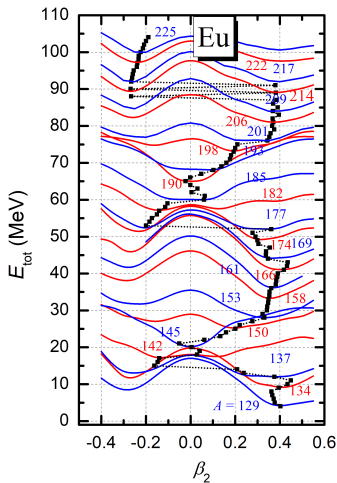


# Deformation $\beta_2$ and potential energy curve: Eu

Deformation for Eu ( $Z = 63$ )

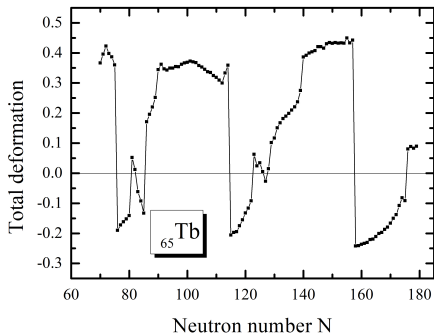


Evolution of potential energy

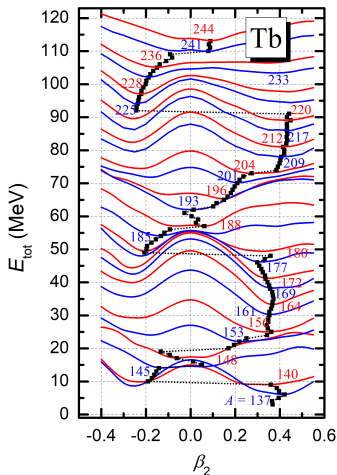


# Deformation $\beta_2$ and potential energy curve: Tb

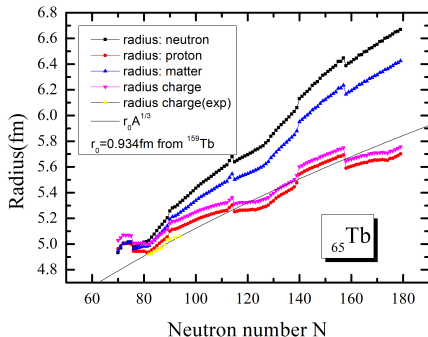
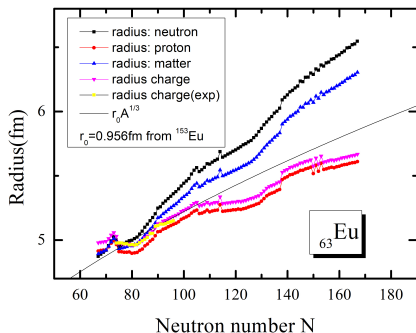
## Deformation for Tb ( $Z = 65$ )



## Evolution of potential energy

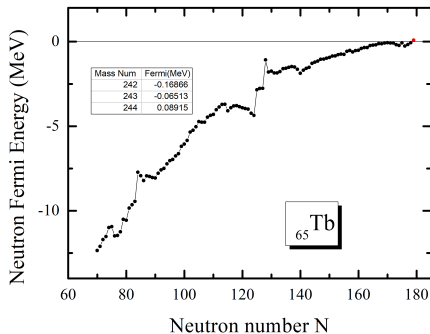
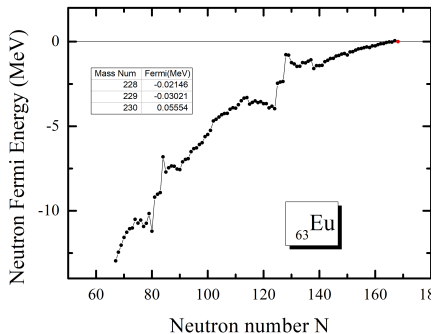


# Radius: Eu ( $Z = 63$ ) and Tb ( $Z = 65$ )

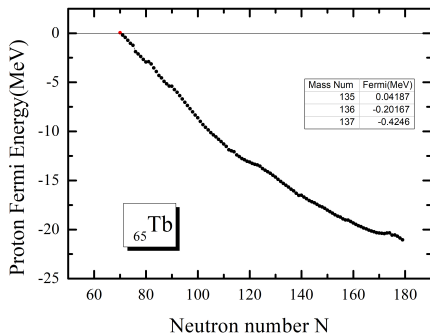
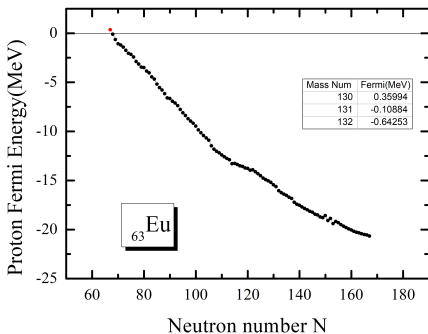


DRHBc overestimates radius for Tb ( $Z = 65$ )

# Neutron Fermi Energy: Eu and Tb



# Proton Fermi Energy: Eu and Tb



Extend the proton drip line from  ${}^{137}\text{Tb}$  to  ${}^{136}\text{Tb}$

# Summary and Future plan

- Calculations completed (Odd-even isotopes)
  - All isotopes completed
- Calculations completed (Odd-odd isotopes)
  - All isotopes completed
- Future calculations
  - Constrained and unconstrained calculation for  $Z=129$

# Physics conclusion

- Nuclei mass prediction can be improved by including rotational correction
- Large difference between experimental and DRHBc  $S_{2n}$  near magic number ( $N=82$ )
- There exist bounded odd-odd and odd-even nuclei beyond one-neutron drip line (for Eu)
- Deformation matches with the evolution of potential energy
- DRHBc overestimates radius for Tb
- Odd-odd nuclei can extend the proton (for Tb) drip line