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Prolate dominance in atomic nuclei with the deformed relativistic Hatree-Bogoliubov theory in continuum

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Introduction

- Nuclear shape is one of the fundamental properties of nuclear structure and related to many other properties.

- The ground states of some nuclei are spherical, while others are deformed in the intrinsic frame due to the spontaneous breaking of rotational symmetry.

- Except for doubly-magic nuclei, most nuclei in the nuclear chart deviate from spherical shape. The observed dominance of prolate over oblate shape is indeed overwhelming: Of the 98 known deformed even-even nuclei identified, only one (^{12}C) is oblate.

  A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin, 1975), Vol. II.

- Another interesting characteristic of nuclear ground-state deformation is that these few oblate nuclei mainly appear near the very end of a major shell.

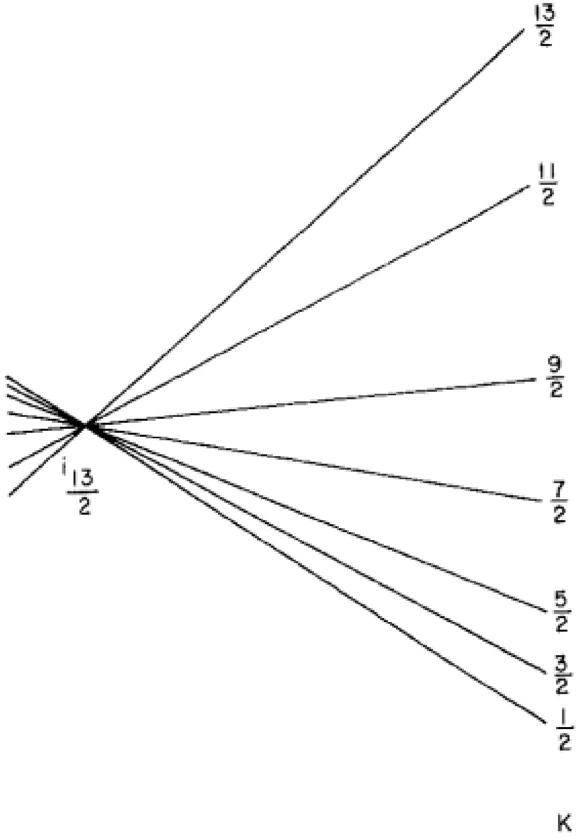
Introduction

- Casten outlined an inspiring understanding on the origin of the prolate-shape dominance from the simple perspective of single particle level diagram.

  - There are more downsloping orbitals with low *K* values on the prolate side than the downsloping orbitals with high *K* values on the oblate side.

  - The close orbits with the same *K* value from different *j* shells on the prolate side will interact which make the lowest orbits be pushed lower in energy and thus gives an energetic advantage to prolate deformations.

  - The lowest orbits after a shell closure often have high *j*, with many *K* values which make a preference for prolate deformations can develop. It's the reason why oblate nuclei are those near the very end of a major shell.



R.F. Casten, *Nuclear Structure From a Simple Perspective* (1990)

Introduction

- To address the origin of the prolate-shape dominance more quantitatively, many different approaches have been employed.

  - Systematic SHF+BCS calculations have studied the prolate shape dominance in the nuclear region with $N > 50$, which was suggested owing to the change of the nature of the major shells from the HO shell to the Mayer-Jensen shell.

    N. Tajima, S. Takahara, and N. Onishi, Nucl. Phys. A **603**, 23 (1996).

  - By comparing the system's total energies defined as the sum of the lowest-lying single-particle energies obtained from pure HO and spheroidal infinite-well potentials, emphasized the importance of the surface of one-body potentials and commented that the spin-orbit potential alone cannot affect the prolate-shape dominance.

    I. Hamamoto and B. R. Mottelson, Phys. Rev. C **79**, 034317 (2009).

  - Within the Nilsson (or Woods-Saxon) Strutinsky method, it is found that the synergism of the surface effect and the spin-orbit potential has an impact on the prolate-shape dominance.

    N. Tajima and N. Suzuki, Phys. Rev. C **64**, 037301 (2001).
    S. Takahara, N. Onishi, Y. R. Shimizu, and N. Tajima, Phys. Lett. B **702**, 429 (2011).

  - The prolate-dominance can also be understood within the algebraic proxy-SU(3) model.

    M. Sugawara, Phys. Rev. C **106**, 024301 (2022).

Introduction

- Pairing correlations are of significance in the description of nuclear ground-state properties.

  - From a general point of view, pairing correlations would enhance the dominance of prolate over oblate, as oblate minima with relatively smaller deformations more easily become spherical under pairing correlations.

    I. Hamamoto and B. R. Mottelson, Phys. Rev. C **79**, 034317 (2009).

  - However, the numerical results based on the Strutinsky shell-correction method showed that pairing correlations have different influences under different parameter conditions.

  - In some cases pairing correlations may enhance prolate-shape dominance whereas in others they may have preference for oblate ones.

    N. Tajima, Y. R. Shimizu, and N. Suzuki, Prog. Theor. Phys. Suppl. **146**, 628 (2002).
    S. Takahara, N. Tajima, and Y. R. Shimizu, Phys. Rev. C **86**, 064323 (2012).

Introduction

- Based on CDFT and simultaneously considering the pairing, continuum, and deformation effects, the DRHBc theory can treat the stable and exotic nuclei in a unified manner and have achieved great success.

S.-G. Zhou, J. Meng, P. Ring, and E.-G. Zhao, Phys. Rev. C **82**, 011301(R) (2010).
L. Li, J. Meng, P. Ring, E.-G. Zhao, and S.-G. Zhou, Phys. Rev. C **85**, 024312 (2012).

- From the DRHBc mass table, one can easily find the dominance of the prolate shape over the oblate shape.

K.-Y. Zhang et al. (DRHBc Mass Table Collaboration) ADNDT **144** (2022) 101488
P. Guo et al. (DRHBc Mass Table Collaboration) ADNDT **158**, 101661 (2024).

- In this work, we investigate the prolate-oblate competition of deformed nuclei with the DRHBc theory, by taking Te, Xe and Ba isotopes ($82 \leq N \leq 126$) and U, Pu and Cm isotopes ($126 \leq N \leq 258$) as examples.

- In particular, the influence of pairing correlations on the shape competition will be addressed.

Theoretical framework

- The RHB equations for the nucleons read

$$\begin{pmatrix} h_D - \lambda_\tau & \Delta \\ -\Delta^* & -h_D^* + \lambda_\tau \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}.$$

H. Kucharek, P. Ring, Z. Phys. A **339**, 23 (1991)

- For an axially deformed nucleus with spatial reflection symmetry, the potentials and densities can be expanded in terms of Legendre polynomials:

$$f(\boldsymbol{r}) = \sum_\lambda f_\lambda(r) P_\lambda(\cos\theta), \quad \lambda = 0, 2, 4, \ldots$$

- In order to take into account the continuum effect properly, in the DRHBc theory, the RHB equations are solved in a spherical Dirac Woods-Saxon basis.

L.-L. Li, J. Meng, P. Ring, E.-G. Zhao, and S.-G. Zhou, Phys. Rev. C **85**, 024312, (2012)

Theoretical framework

- The total energy of a nucleus is

$$
\begin{aligned}
E_{\mathrm{RHB}} = & \sum_{k>0}(\lambda_\tau - E_k)v_k^2 - E_{\mathrm{pair}} \\
& - \int d^3\boldsymbol{r}\Big(\frac{1}{2}\alpha_S\rho_S^2 + \frac{1}{2}\alpha_V\rho_V^2 + \frac{1}{2}\alpha_{TV}\rho_3^2 \\
& + \frac{2}{3}\beta_S\rho_S^3 + \frac{3}{4}\gamma_S\rho_S^4 + \frac{3}{4}\gamma_V\rho_V^4 + \frac{1}{2}\delta_S\rho_S\Delta\rho_S \\
& + \frac{1}{2}\delta_V\rho_V\Delta\rho_V + \frac{1}{2}\delta_{TV}\rho_3\Delta\rho_3 + \frac{1}{2}\rho_p e A^0\Big) + E_{\mathrm{c.m.}},
\end{aligned}
$$

- The quadrupole deformation is calculated by

$$
\beta_{\tau,2} = \frac{\sqrt{5\pi}\,Q_{\tau,2}}{3N_\tau\langle r_\tau^2\rangle}.
$$

L.-L. Li, J. Meng, P. Ring, E.-G. Zhao, and S.-G. Zhou, Phys. Rev. C **85**, 024312, (2012)

Numerical details

■ DRHBc calculations

- Six isotopic chains: even-Z Te, Xe, Ba ($82 \leq N \leq 126$)

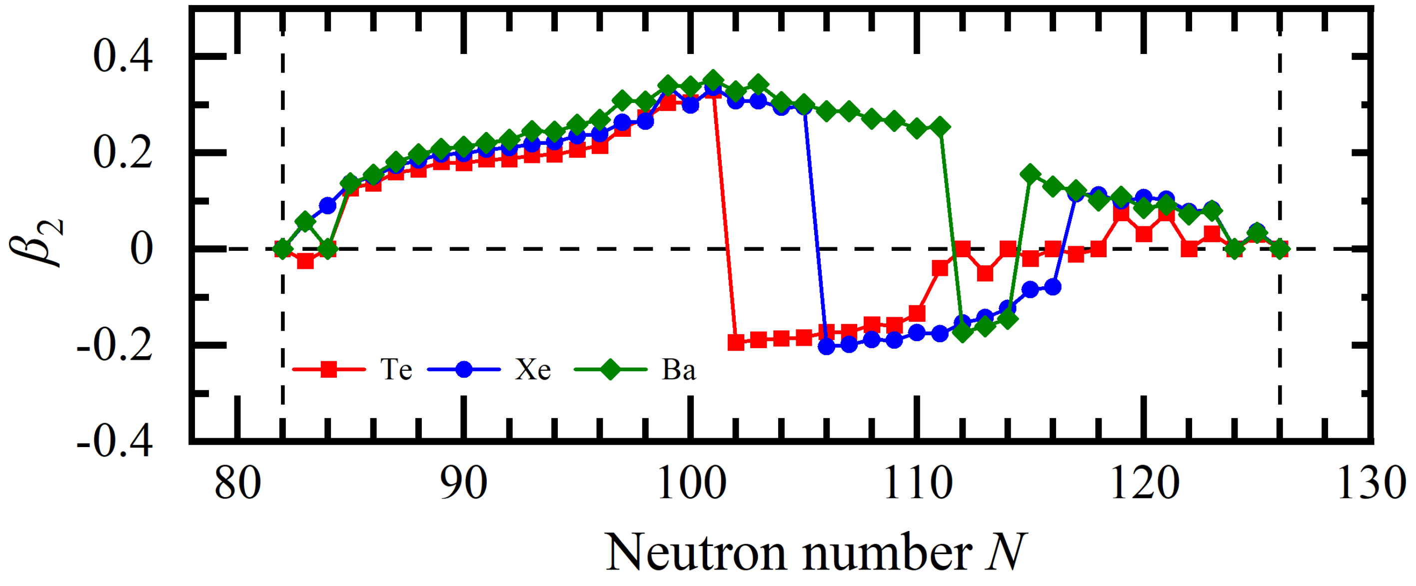
  even-even U, Pu, Cm ($126 \leq N \leq 184$; $184 \leq N \leq 258$)

- Density functional: PC-PK1
- Pairing strength: $V_0 = -325$ MeV fm^3
- Saturation density: $\rho_{\mathrm{sat}} = 0.152$ fm^{-3}
- Pairing window: 100 MeV
- Energy cutoff: $E_{\mathrm{cut}} = 300$ MeV
- Angular momentum cutoff: $J_{max} = 23/2\,\hbar$
- Legendre expansion truncation: $\lambda_{max} = 6$ $(8 \leq Z \leq 70)$; $\lambda_{max} = 8$ $(72 \leq Z \leq 100)$

K.-Y. Zhang et al. (DRHBc Mass Table Collaboration) Phys. Rev. C **102**, 024314, (2020)

Results and discussion: Te, Xe, and Ba isotopes ($82 \le N \le 126$)

■ Evolution of ground-state deformation

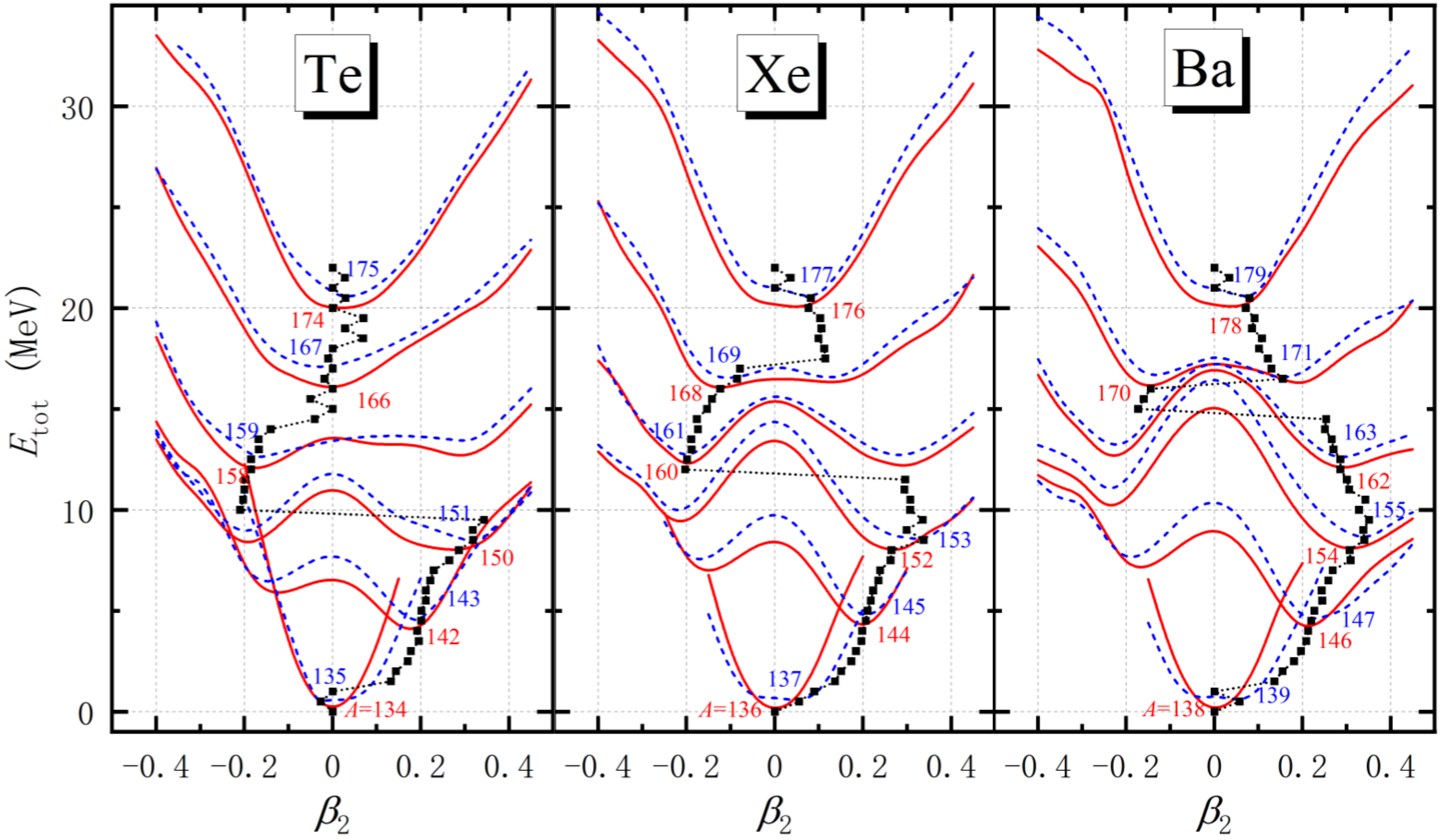


82≤N≤126	Num of def. nuclei	Num of prolate	Percentage
Tellurium	**36**	**22**	**61.1%**
Xenon	**42**	**31**	**73.8%**
Barium	**41**	**38**	**92.7%**

- Sudden changes of ground-state deformation from prolate to oblate are found.
- Prolate shape dominance is obviously seen.
- As Z increases from 52 to 56, the transformation point changes (**delay/hasten**). As a result, the prolate shape becomes more dominant.

P. Guo, C. Pan, Y. C. Zhao, X. K. Du, and S. Q. Zhang, Phys. Rev. C **108**, 014319 (2023)

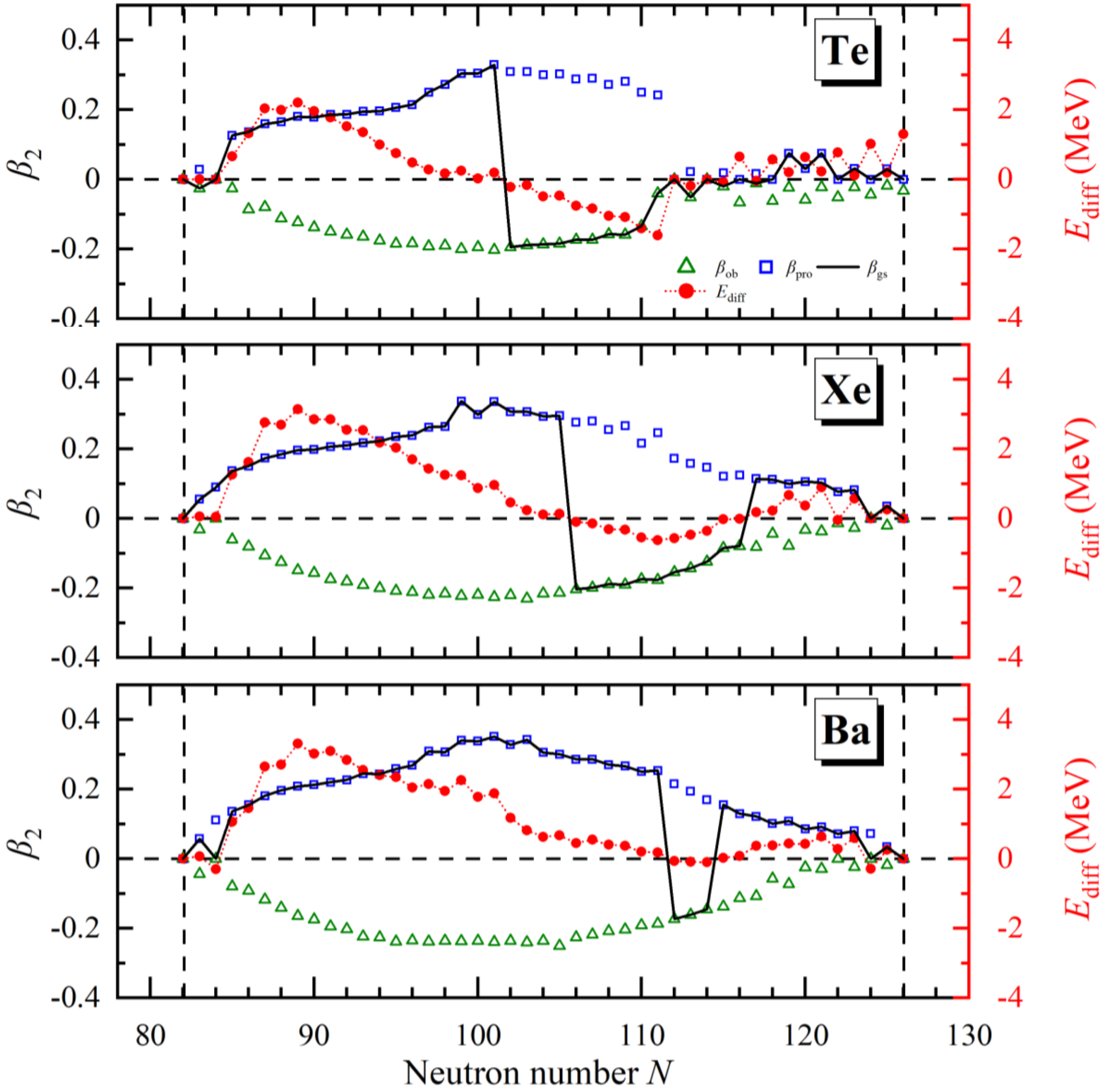
PECs of Te, Xe and Ba isotopes ($82 \leq N \leq 126$)



- Most nuclei have two local minima with one at the prolate side and another at the oblate side.
- The sudden change of ground-state deformation occurs where two local minima have similar energies.

Shape coexistence ⇨ **"Prolate-oblate competition"** ⇨ **Ground state**

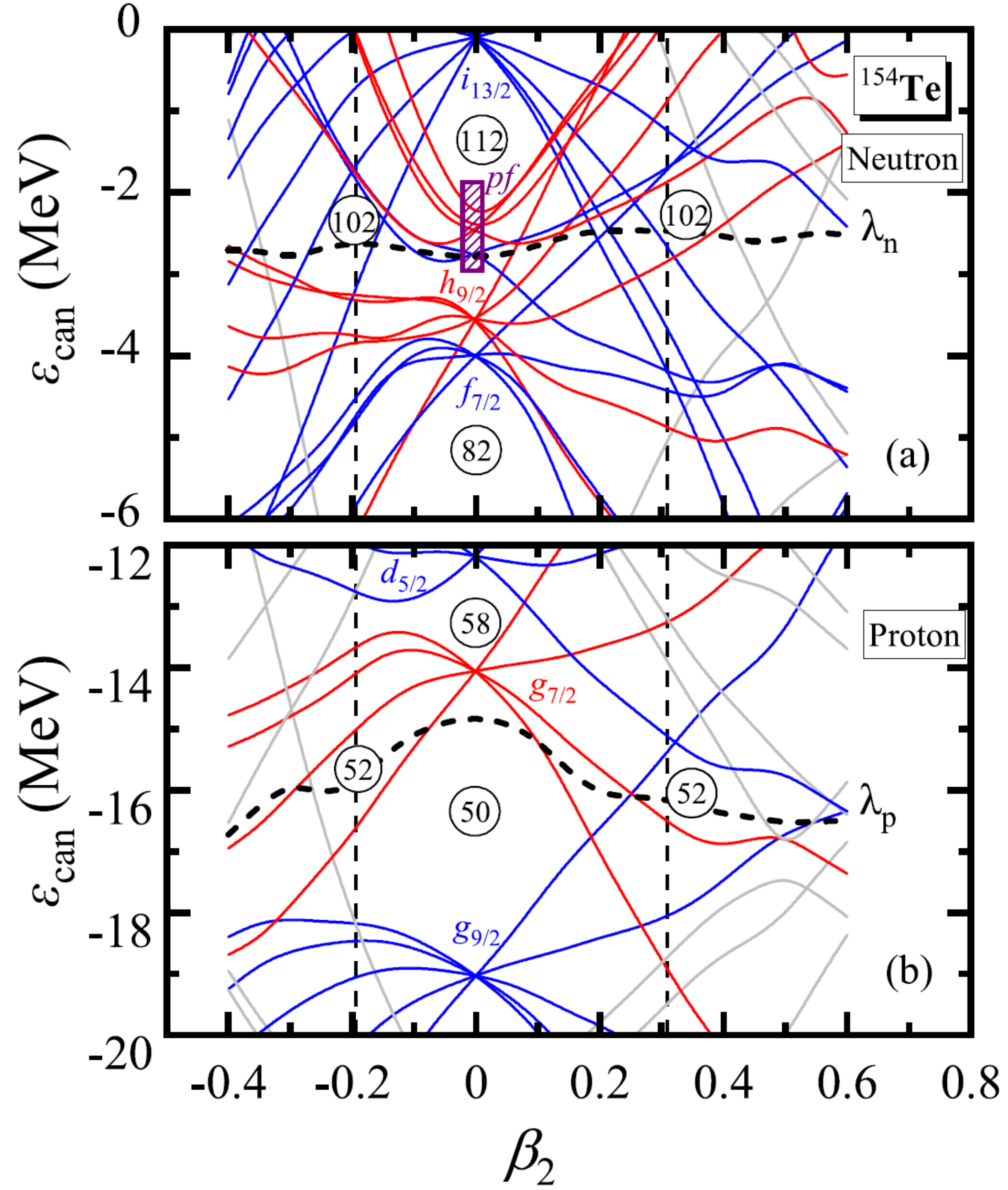
■ Evolution of energy difference between minima E_{diff}



- The sudden change of ground-state deformation occurs where E_{diff} changes the sign.
- From Te to Ba, the curve of E_{diff} shows an upward trend, which corresponds to the delay of the first transformation point.
- The increase of Z from 52 to 56 enhances the prolate-shape dominance.

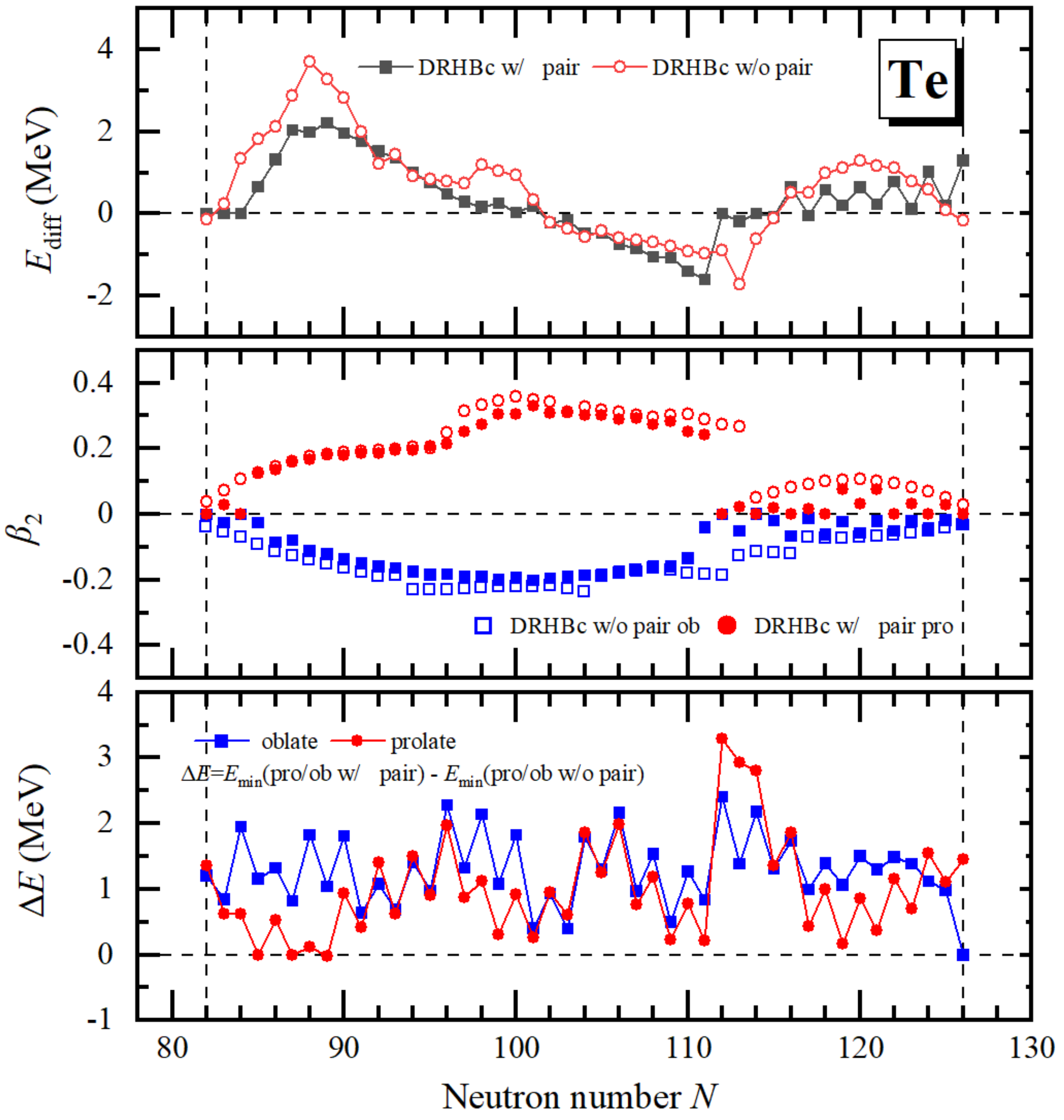
E_{diff}=E_{ob}(the oblate minimum) − E_{pro} (the prolate minimum)

■ Single particle levels of ^{154}Te



- Similar to the discussion by Casten, the prolate shape dominance can be understood by the single neutron levels obtained in the DRHBc theory.
  - After the shell closure (*N*=82), the lowest two levels are $f_{7/2}$ and $h_{9/2}$ which have relatively high *j*, with many *K* values, and thus the prolate side orbits with the same *K* value from different *j* shells will be close together.
  - They will interact and the lowest orbits will be pushed lower in energy which gives an energetic advantage to prolate deformations.
  - Similarly, the filling of protons in the downsloping g7/2 orbital after the shell closure (Z = 50) can also bring more energetic preference for prolate shape.

Influence of pairing correlations

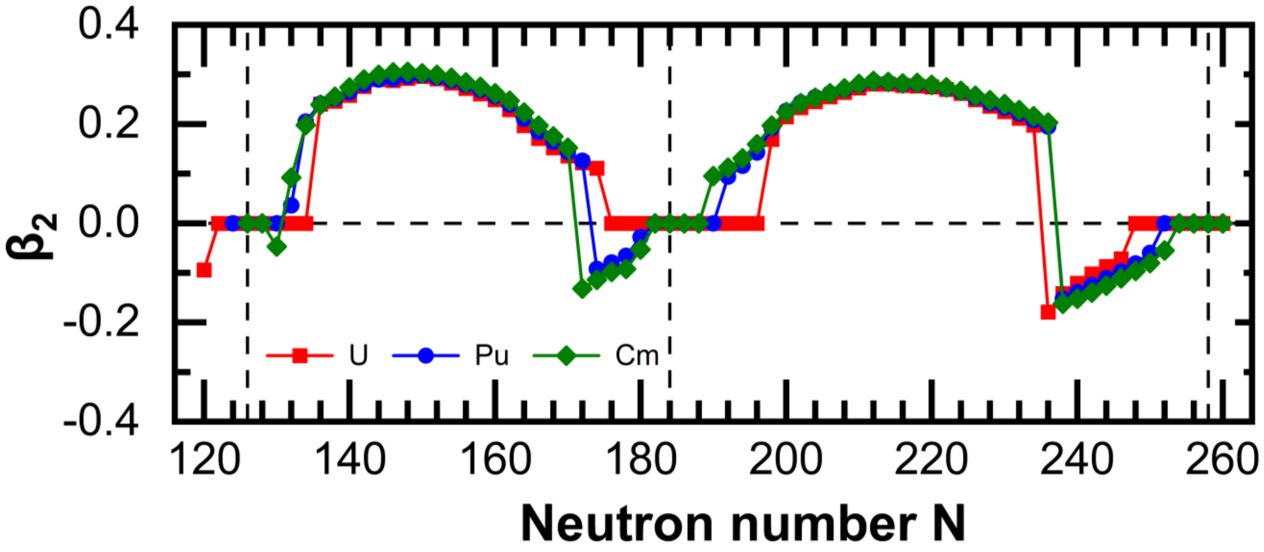


$\Delta E = E_{\min}$(prolate/oblate without pair) $- E_{\min}$ (prolate/oblate with pair)

- Pairing correlations tend to make E_{diff} smaller in most region of the isotopic chain.
- Pairing correlations tend to make β_2 smaller.
- Pairing correlations tend to make both local minima more bound and have larger influence on the oblate one, but have little impact on the prolate shape dominance.

P. Guo, C. Pan, Y. C. Zhao, X. K. Du, and S. Q. Zhang, Phys. Rev. C **108**, 014319 (2023)

Results and discussion: U, Pu, and Cm isotopes ($126 \leq N \leq 258$)

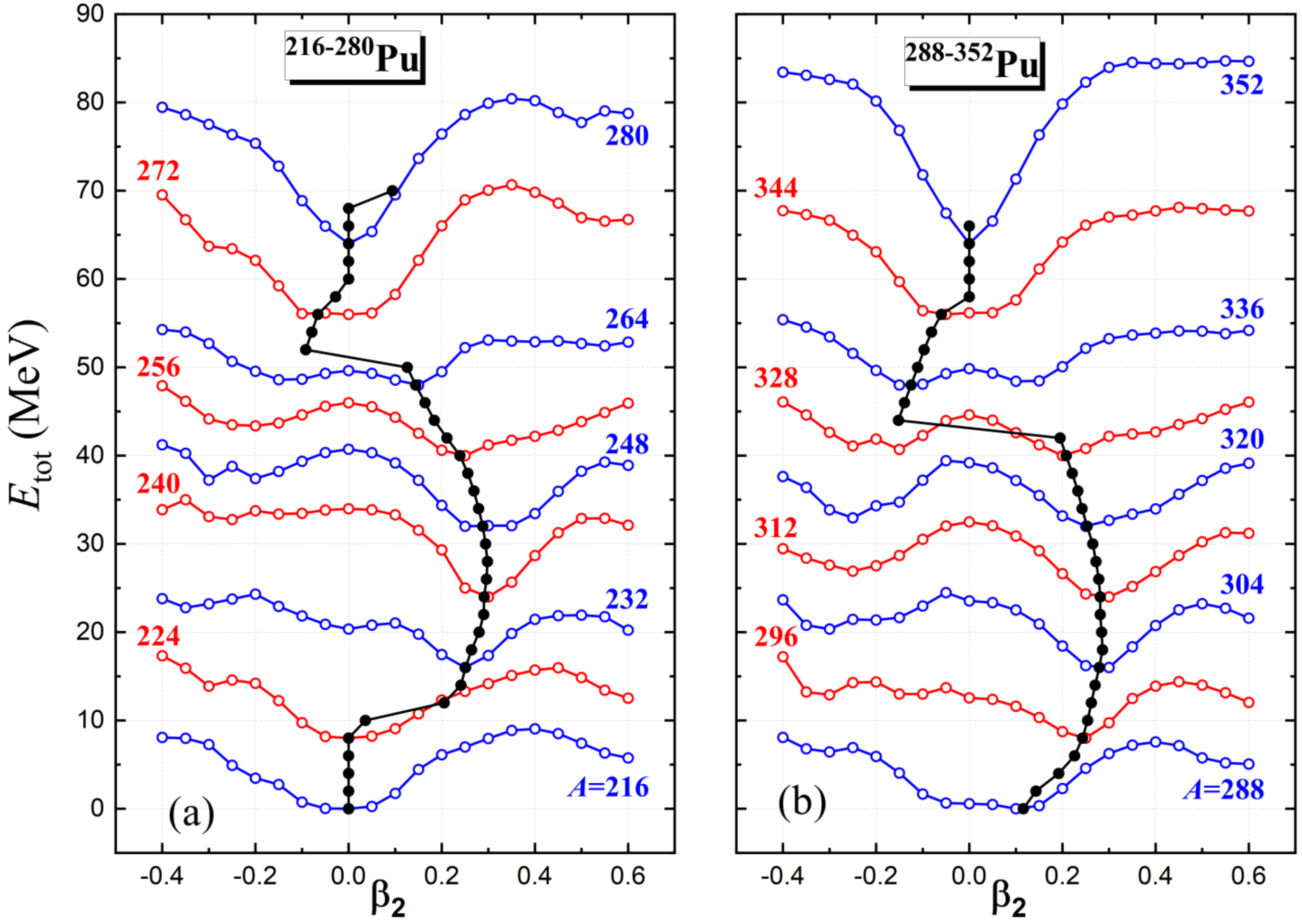


- G.S. shape evolution: "Periodic-like behavior"
- Prolate shape dominance can be seen in both periods.
- From U to Cm, the prolate shape becomes somewhat less dominant. The behavior is **opposite** to the region of Te to Ba.

$126 \leq N \leq 184$	Deformed	Prolate	Percentage
Uranium	**21**	**20**	**95.24%**
Plutonium	**25**	**21**	**84.00%**
Curium	**26**	**20**	**76.92%**
$184 \leq N \leq 258$	Deformed	Prolate	Percentage
Uranium	**25**	**19**	**76.00%**
Plutonium	**30**	**23**	**76.67%**
Curium	**32**	**24**	**75.00%**
Total	Deformed	Prolate	Percentage
Uranium	**46**	**39**	**84.78%**
Plutonium	**55**	**44**	**80.00%**
Curium	**58**	**44**	**75.86%**

PECs of $^{216\text{-}352}$Pu

$122 \leq N \leq 186$ $194 \leq N \leq 258$

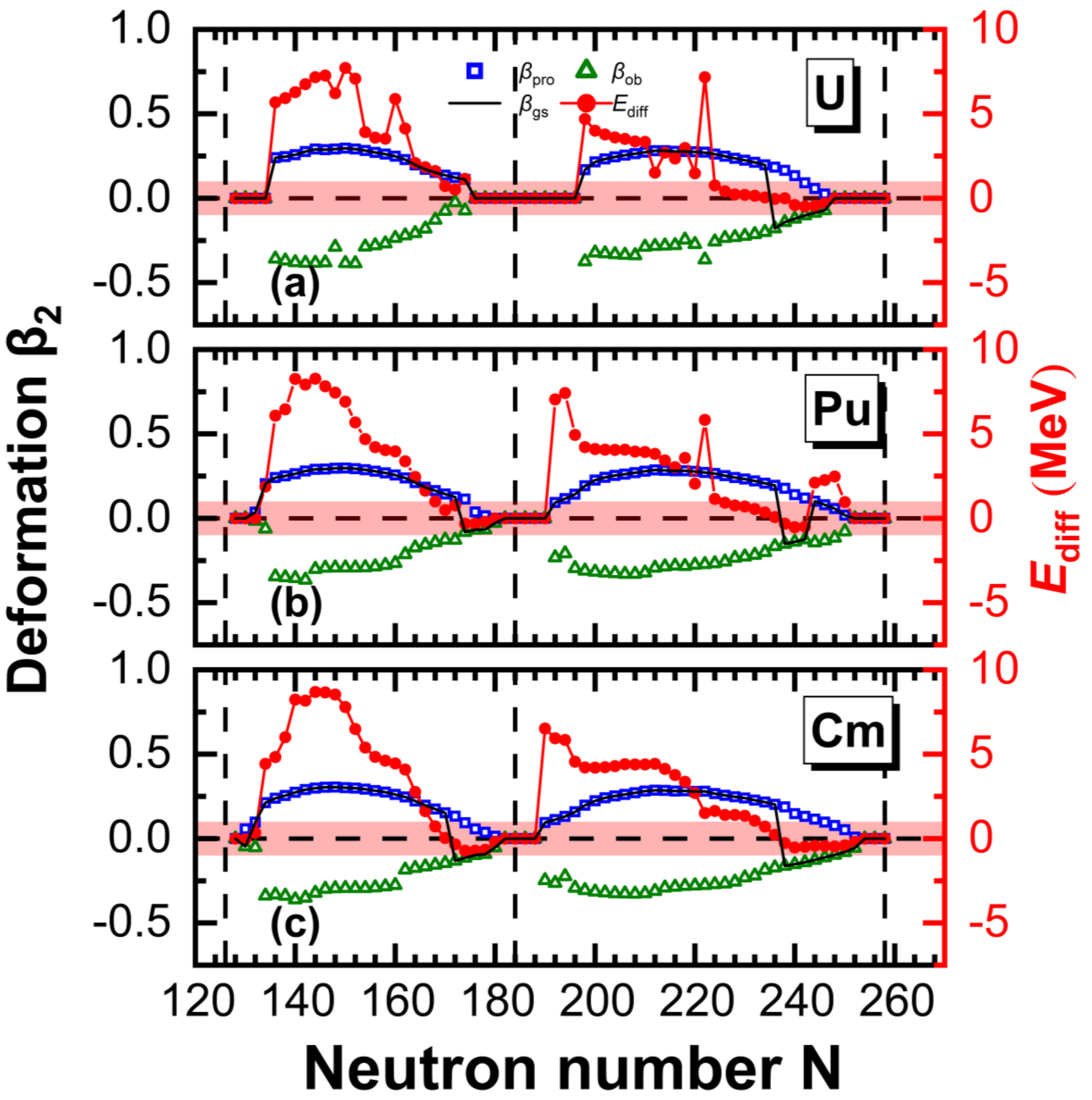


- Most nuclei have two local minima with one at the prolate side and another at the oblate side.

- Sudden changes from prolate to oblate shapes exist in both periods: occurs where two local minima have similar energies.

"Prolate-oblate competition"

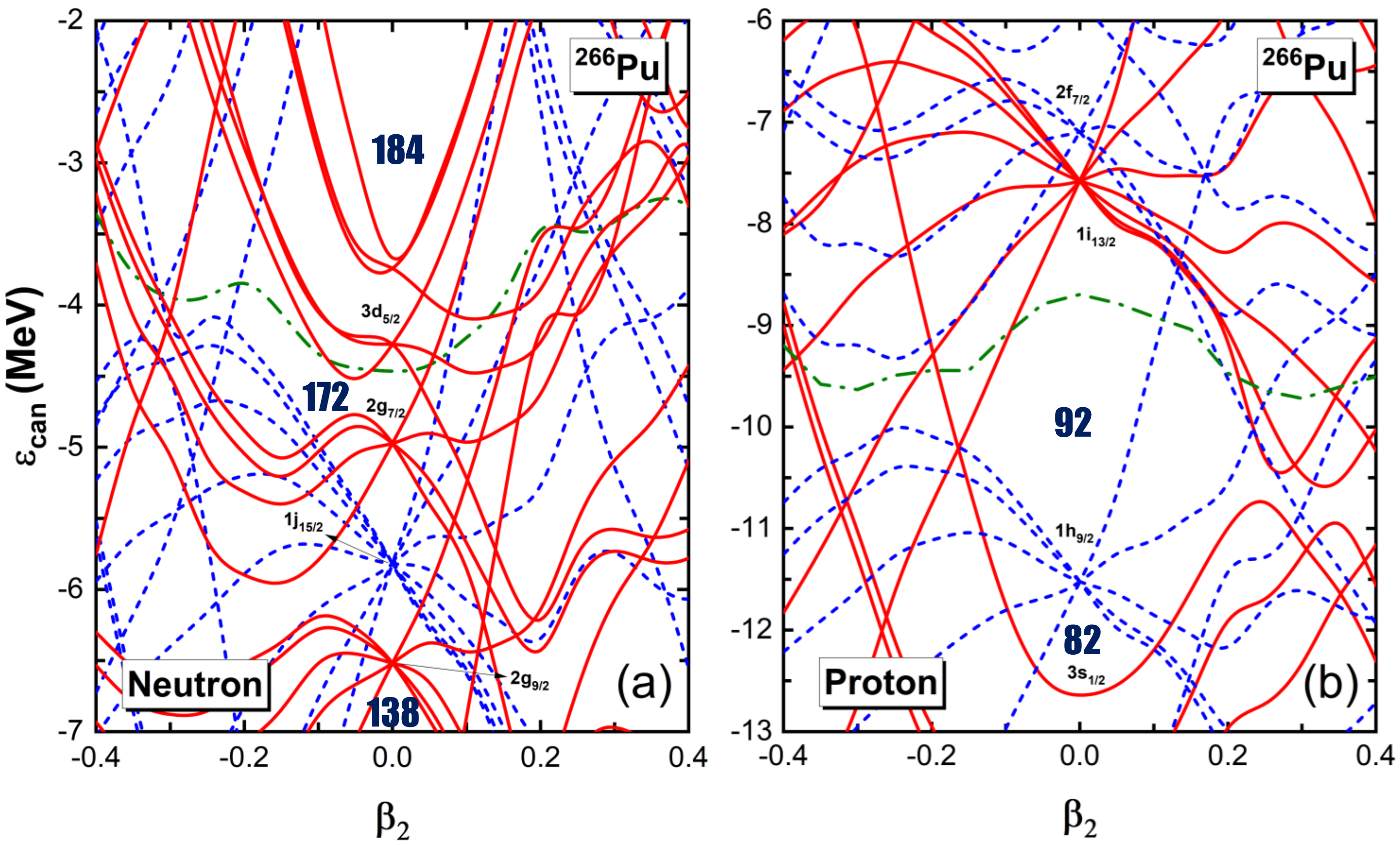
■ The evolution of the E_{diff}



- Both prolate and oblate deformations, as well as their E_{diff} evolve similarly for U, Pu, and Cm.
- The sudden change of ground-state deformation occurs where E_{diff} changes the sign.
- From U to Cm, the curve of E_{diff} shows an upward trend in the former part but a downward trend in the latter part, indicating a delicate dependence on nucleon numbers.

$E_{\text{diff}}=E_{\text{ob}}$(the oblate minimum) $-$ E_{pro} (the prolate minimum)

■ Single particle levels of transformation point nucleus $^{266}_{94}Pu_{172}$



- After the shell closure (N=126), the lowest levels are $1i_{11/2}$, $2g_{9/2}$, $1j_{15/2}$ and $2g_{7/2}$ which have relatively high j, with many K values, and thus the prolate side orbits with the same K value from different j shells will be close together.

- They will interact and the lowest orbits will be pushed lower in energy which gives an energetic advantage to prolate deformations.

- For proton, the appearance of pseudo-shell Z=92 may affect the prolate-oblate competition.

Summary and Perspective

- Most nuclei have two local minima with one at the prolate side and another at the oblate side. The prolate-oblate competition comes from the competition of the two local minima's energy.

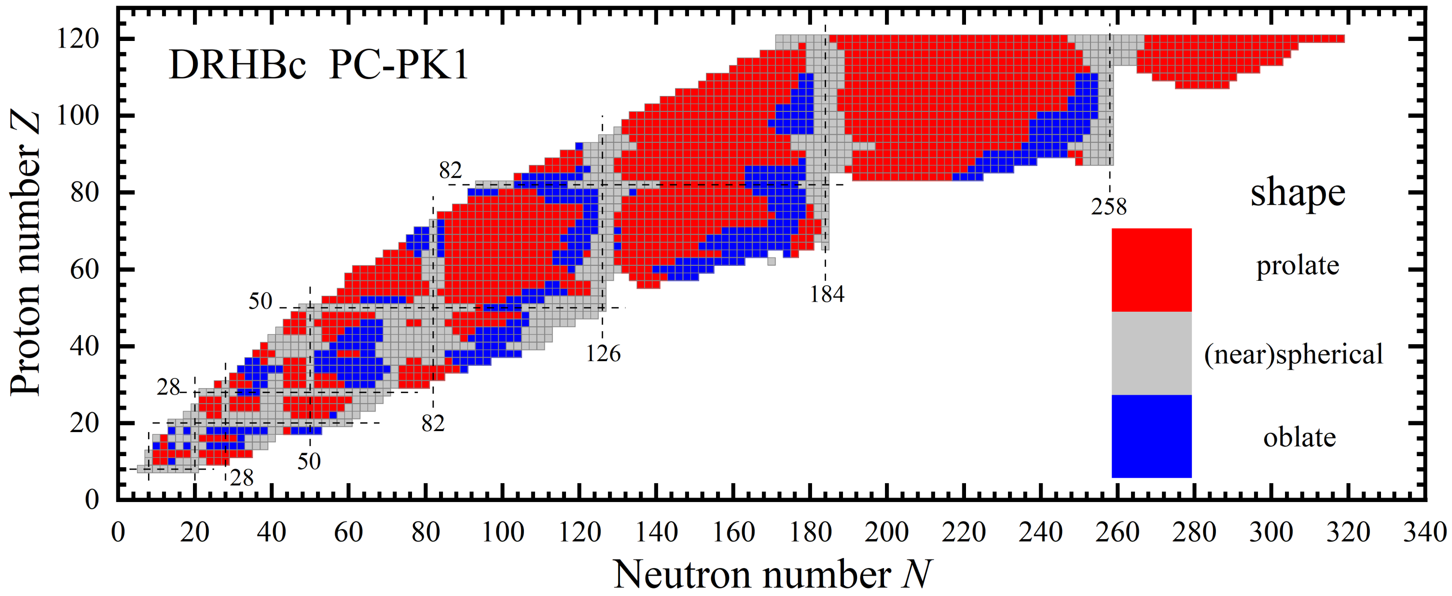
- The prolate advantage and sudden change to the oblate deformation can be understood by the single neutron levels obtained by DRHBc theory.

- Pairing correlations will make nuclei more bound and reduce the value of β_2. It has a larger influence on the local minimum at the oblate side but has little impact on the prolate shape dominance.

- With the DRHBc mass table, we can investigate the prolate-shape dominance on the whole nuclear chart, in particular for the vast weakly bound nuclei.

- To study the triaxial effects on the prolate-shape dominance by using the TRHBc and to explore the beyond-mean-field effects by restoring the rotational invariance and considering the shape fluctuations are interesting.

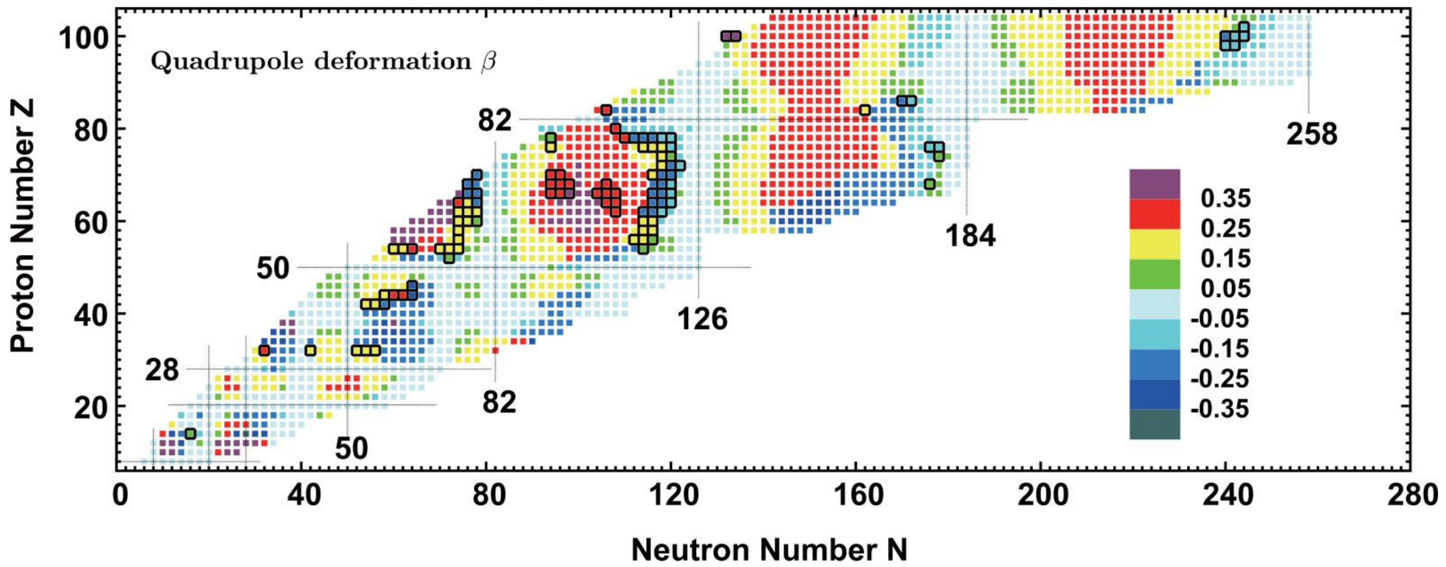
Summary and Perspective



DRHBc + PC-PK1

Courtesy of Peng Guo

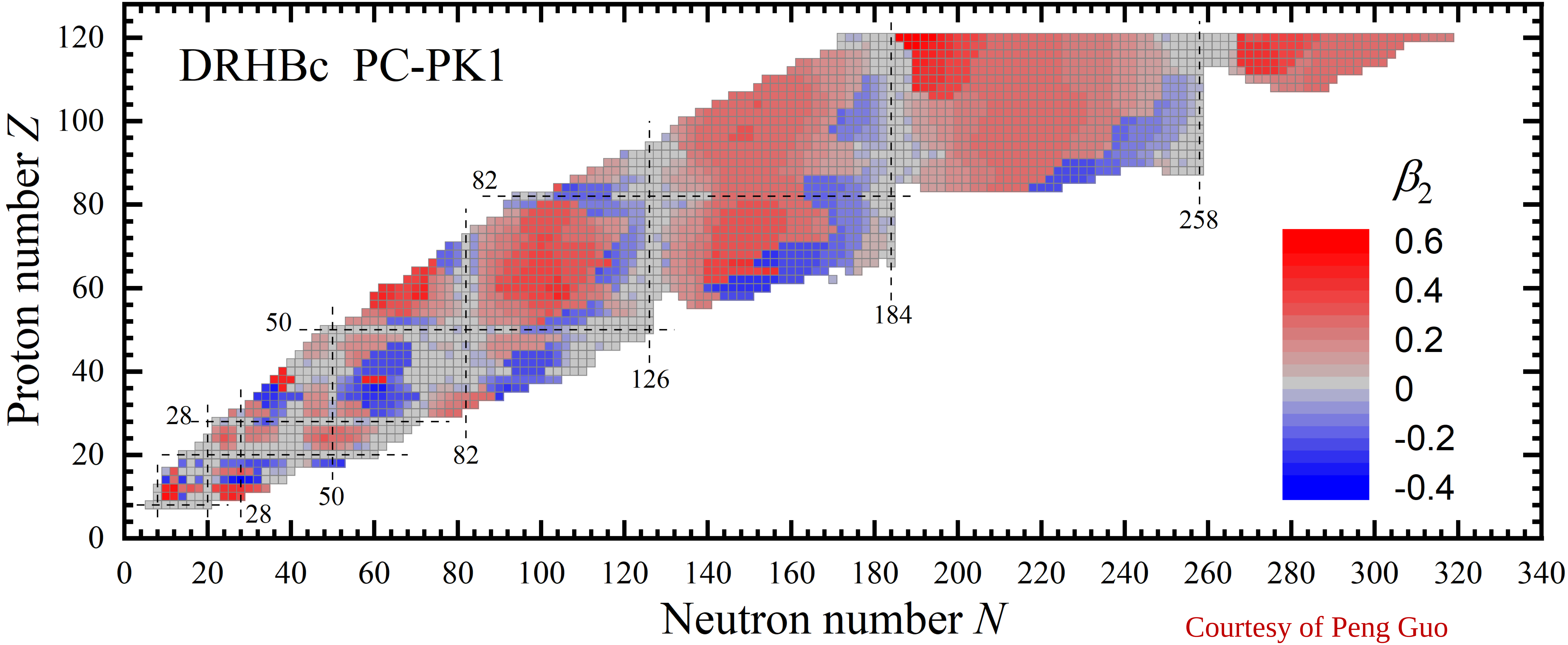
K.-Y. Zhang et al. (DRHBc Mass Table Collaboration) At. Data and Nucl. Data Tables 144, 101488 (2022)



Tri RHB + PC-PK1

Y. L. Yang, Y. K. Wang, P. W. Zhao, and Z. P. Li, Phys. Rev. C 104, 054312 (2021).

Thank you for your attention!



Courtesy of Peng Guo