蘭州大學
LANZHOU UNIVERSITY

The 7th International Workshop on DRHBc Mass Table
1-4 July 2024
Gangneung Green City, Gangneung-si, Korea.

Finite-Amplitude Method for Charge-Exchange Transitions in Axially Deformed Nuclei based on Relativistic Energy Density Functional Theory

Chen Chen

Supervisor：Prof. Yifei Niu

School of Nuclear Science and Technology
Lanzhou University

7/16/2024

Outline

- Introduction
- Theoretical Framework
- Numerical Details
- Preliminary Results
- Summary and Outlook

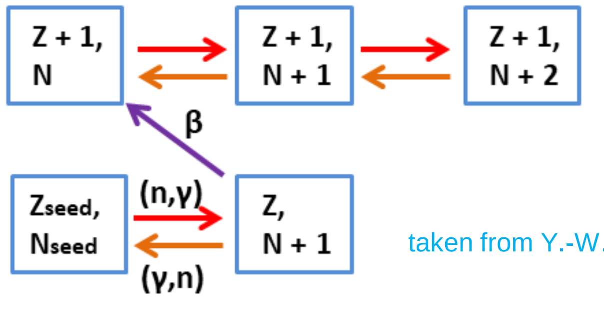
Introduction

➢ Collective vibrations are one of the hot topics in nuclear physics and astrophysics.

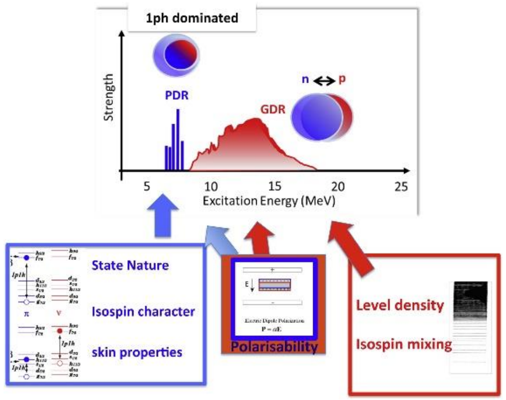
- **Charge-exchange** modes: Isobaric Analog Resonance (IAR), Gamow-Teller Resonances (GTR) ……
  - *β*-decay rates in *r*-process
    T. Kajino et al. Prog. Part. Nucl. Phys. 107, 109 (2019).
  - nuclear matrix elements of the neutrinoless double *β*-decay
    J. M. Yao et al. Prog. Part. Nucl. Phys. 126, 103965 (2022).
- **Non-charge-exchange** modes: Giant Dipole Resonances (GDR), Pygmy Dipole Resonances (PDR) ……
  - neutron capture rates in *r*-process
  - neutron skin thickness & symmetry energy parameters
    X. Roca-Maza et al. Prog. Part. Nucl. Phys. 101, 96 (2018).

It's important to improve the understanding of collective vibrations!

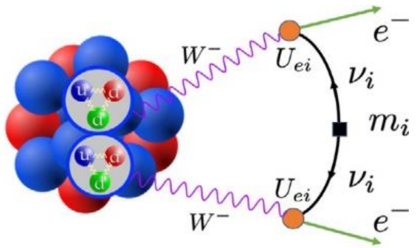
r-process



taken from Y.-W. Hao



A. Bracco et al. Prog. Part. Nucl. Phys. 106, 360 (2019).

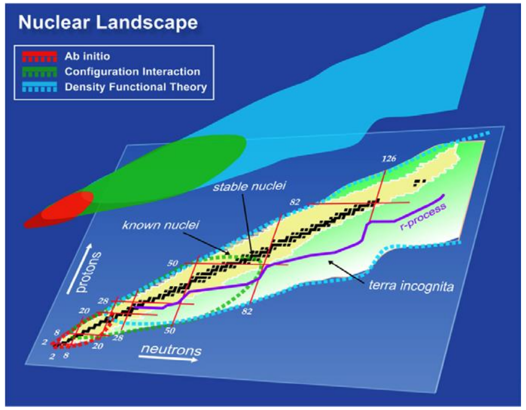


J. M. Yao et al. Prog. Part. Nucl. Phys. 126, 103965 (2022).

Introduction

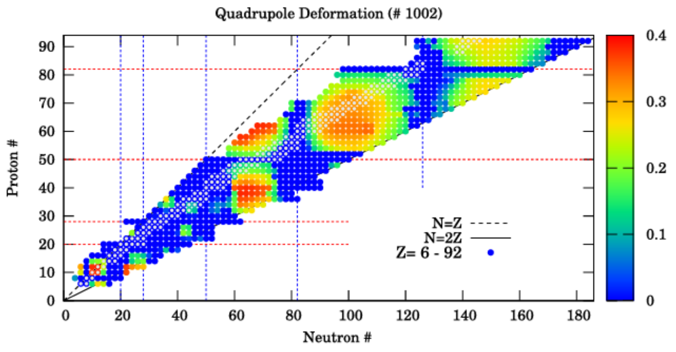
- Theoretical descriptions:
  - *ab initio* approach
    J. Birkhan et al. Phys. Rev. Lett. 118, 252501 (2017).
  - shell model
    E. Caurier et al. Rev. Mod. Phys. 77, 427 (2005).
  - quasiparticle random-phase approximation (QRPA) based on density functional theory (DFT);
    N. Paar et al. Rep. Prog. Phys. 70, 691–793 (2007).

  QRPA is the most efficient method for global descriptions, but most applications are under spherical approximation.



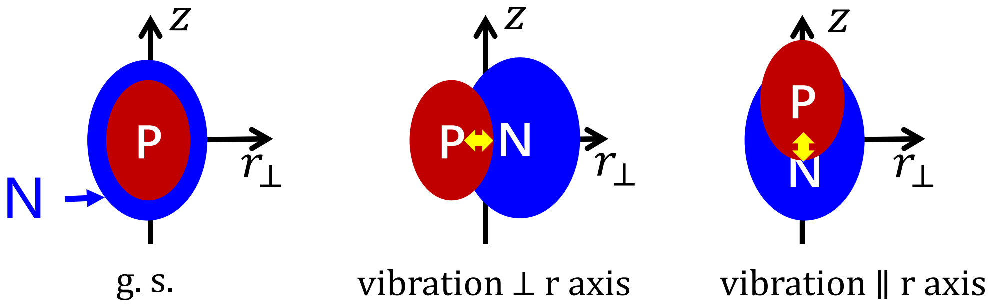
R. J. Furnstahl et al. Rep. Prog. Phys. 76, 126301 (2013).

- The deformation is necessary for global descriptions.
  - Many nuclei are deformed;



S. Ebata et al. Phys. Scr. 92, 064005 (2017).

  - Nuclear deformation leads to considerable splitting in the transition strength.



Introduction

➢ QRPA for deformed nuclei:

- Matrix diagonalization:

  D. Pena Arteaga et al. Phys. Rev. C 77, 034317 (2008);

  M. T. Mustonen et al. Phys. Rev. C 87, 064302 (2013);

  Experimental data are well-reproduced,

  but the cost of diagonalization increases extremely.

- Quasiparticle finite amplitude method (QFAM):

  T. Nakatsukasa et al. Phys. Rev. C 76, 024318 (2007)

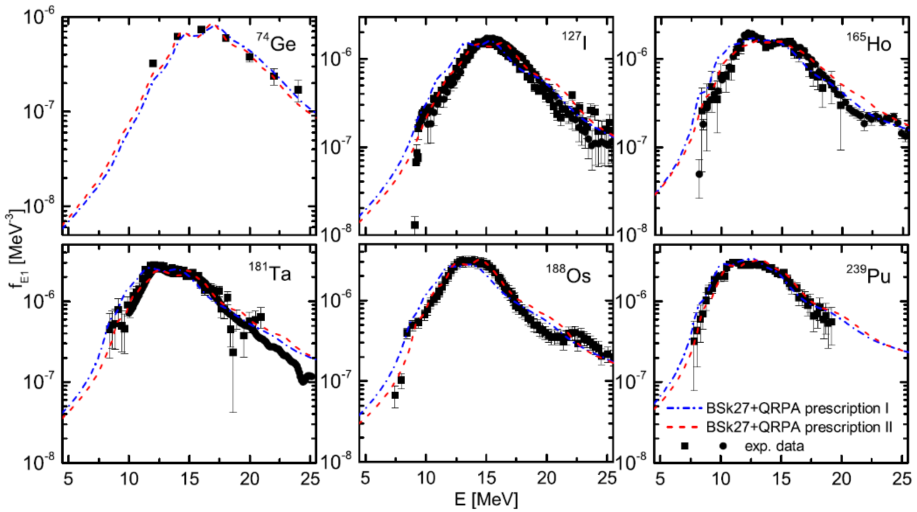
  T. Nikšić et al. Phys. Rev. C 88, 044327 (2013).

  X. W. Sun et al. Phys. Rev. C 96, 024614 (2017).

  A. Bjelčić et al. Comput. Phys. Commun. 253, 107184 (2020).

  Theoretically equivalent to Matrix-QRPA;

  By avoiding the diagonalization routine, it's more efficient for global descriptions.



Y. Xu et al. Phys. Rev. C 104, 044301 (2021).

Introduction

➢ QFAM developments:

- For non-charge-exchange transitions
  Skyrme QFAM: P. Avogadro et al. Phys. Rev. C 84, 014314 (2011) …….
  Relativistic QFAM: T. Nikšić et al. Phys. Rev. C 88, 044327 (2013), X. W. Sun et al. Phys. Rev. C 96, 024614 (2017) …….

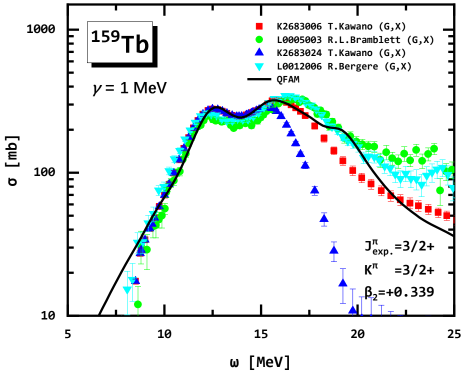
- For charge-exchange transitions
  Skyrme QFAM: M. T. Mustonen et al. Phys. Rev. C 10 (2016)…….

No relativistic QFAM model for charge-exchange transitions.

Although, very recently, a new method adopting liner response theory in terms of separable interactions based on relativistic DFT with speed comparable to QFAM has been introduced. A. Ravlić et al. Preprint at http://arxiv.org/abs/2404.13266 (2024).

➢ Relativistic DFTs preserve the Lorentz invariance and naturally include the spin-orbit interaction.
J. Meng et al. Prog. Part. Nucl. Phys. 57, 470 (2006).

- Based on relativistic QFAM, we have investigated the photon-absorption cross section systemically.



In this work, our aim is to develop a relativistic QFAM model for charge-exchange transitions

Theoretical Framework

- Starting from time-dependent Hartree-Fock-Bogoliubov equation under the perturbation $\mathcal{F}(t)$,

$$\mathrm{i}\hbar\partial_t\mathcal{R}(t) = [\mathcal{H}(t) + \mathcal{F}(t), \mathcal{R}(t)\,],$$

- The first-order terms above correspond to the time-dependent linear-response equation,

$$\mathrm{i}\hbar\partial_t\delta\mathcal{R}(t) = [\mathcal{H}_0, \delta\mathcal{R}(t)\,] + [\delta\mathcal{H}(t), \mathcal{R}_0] + [\mathcal{F}(t), \mathcal{R}_0],$$

- By Fourier transformation, linear-response equation can be written in the frequency domain,

$$\omega\delta\mathcal{R}(\omega) = [\mathcal{H}_0, \delta\mathcal{R}(\omega)\,] + [\delta\mathcal{H}(\omega), \mathcal{R}_0] + [\mathcal{F}(\omega), \mathcal{R}_0],$$

Or written in quasiparticle representation ($\delta\mathcal{R}$ is represented as $\mathcal{X}$ and $\mathcal{Y}$), know as QFAM equation,

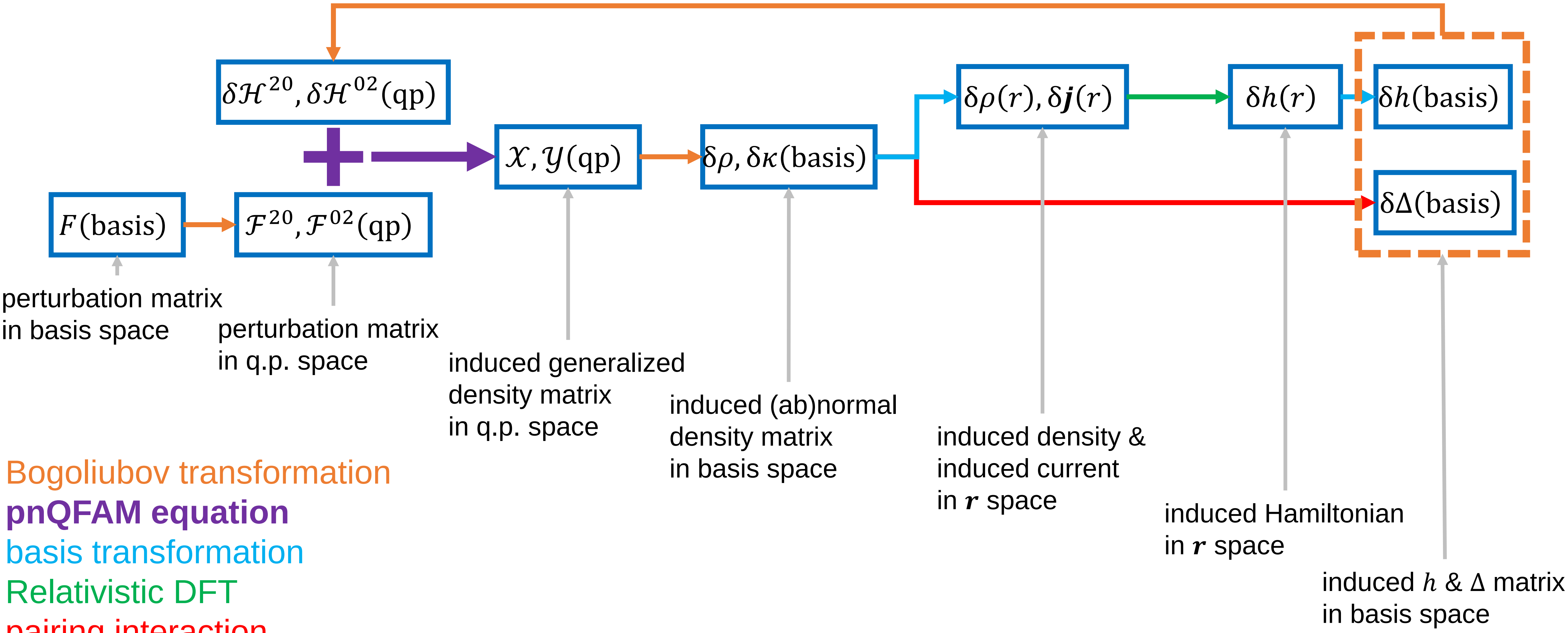
$$\left(E_\mu + E_\nu - \omega\right)\mathcal{X}_{\mu\nu}(\omega) + \delta\mathcal{H}^{20}_{\mu\nu}(\omega) = -\mathcal{F}^{20}_{\mu\nu},$$

$$\left(E_\mu + E_\nu + \omega\right)\mathcal{Y}_{\mu\nu}(\omega) + \delta\mathcal{H}^{02}_{\mu\nu}(\omega) = -\mathcal{F}^{02}_{\mu\nu},$$

For charge-exchange transitions, index μ & ν indicate quasiparticle with different projected isospin.

Theoretical Framework

➢ The flow chart of QFAM equation



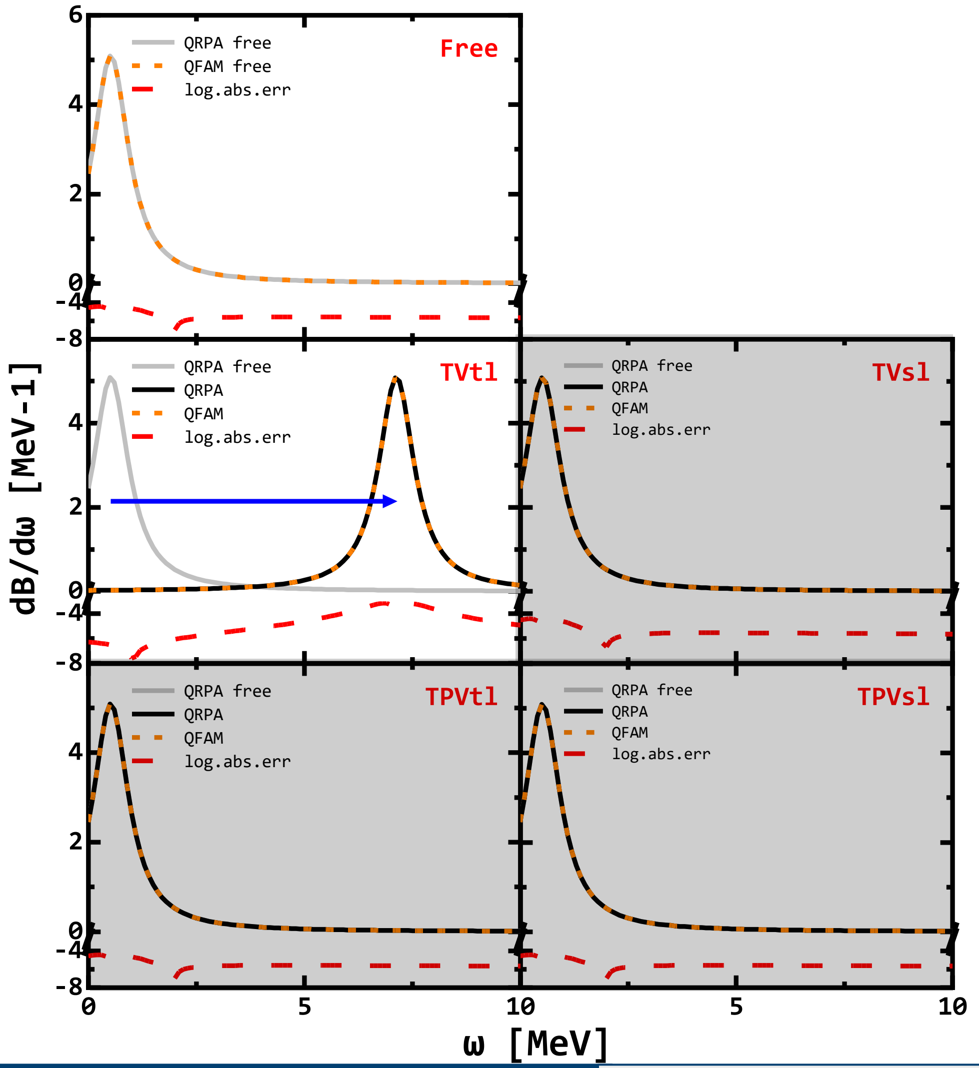
Bogoliubov transformation
pnQFAM equation
basis transformation
Relativistic DFT
pairing interaction

Numerical Details

- p-h interaction: density-dependent point-coupling density functional
  - DD-PC1 + isovector-pseudovector channel $\alpha_{\mathrm{TPV}} = 0.734$;
    T. Nikšić et al. Phys. Rev. C 78, 034318 (2008); D. Vale et al. Phys. Rev. C 103, 064307 (2021).
- p-p interaction: finite range separable pairing interaction (included in RHB, not yet in QFAM)
  - $G = 728\ \mathrm{MeVfm}^3$, $a = 0.644\ \mathrm{fm}$
    Y. Tian et al. Phys. Lett. B 676, 44 (2009).
- Basis expansion: axially deformed harmonic oscillator (ADHO) basis
  - RHB: only $\Omega > 0$;
  - QFAM: $\Omega > 0$ & its time-reversed state $\bar{\Omega}$;
  - Major shell quantum number truncation: $N_{\max} = 14$ for large components, $N_{\max} + 1$ for small components.

Numerical Details

➢ Compared with QRPA under the spherical approximation: ^{48}Ca



- Fermi transition (natural parity, $J^\pi = 0^+$, $K = 0$):
  - Contributions of each channel:

	isovector-vector (TV)	isovector-pseudovector (TPV)
time-like (tl)	TVtl √	TPVtl ×
space-like (sl)	TVsl ×	TPVsl ×

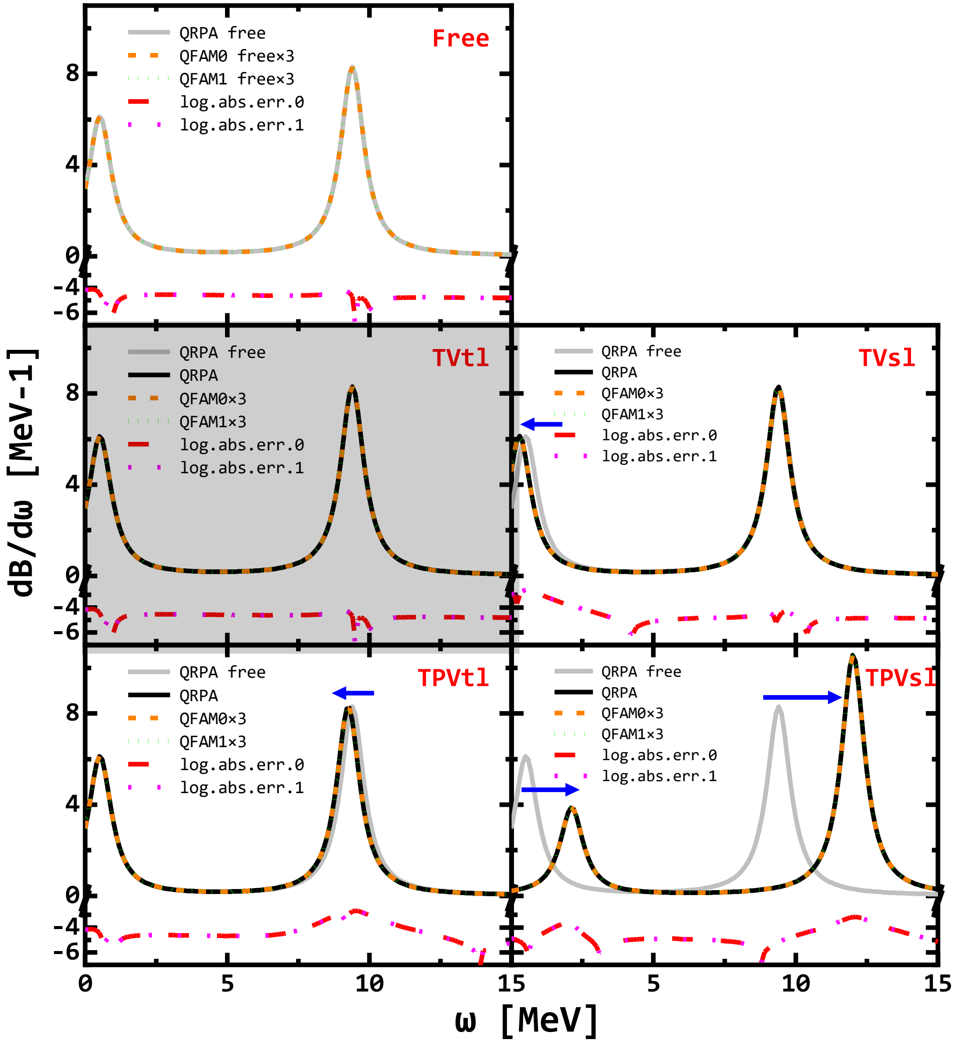
only the TVtl part contributes;

  - The numerical results are consistent with above analysis;

* numerical check are truncated by $N_{\rm max} = 4$ for the calculation convenience.

Numerical Details

➢ Compared with QRPA under the spherical approximation: ^{48}Ca



- Gamow-Teller transition (Non-natural parity, $J^{\pi} = 1^{+}, K = 0,1$):
  - Contributions of each channel:

	isovector-vector (TV)	isovector-pseudovector (TPV)
time-like (tl)	TVtl ×	TPVtl √
space-like (sl)	TVsl √	TPVsl √

TVsl,TPVtl & TPVsl parts contribute;

  - The numerical results are consistent with above analysis;

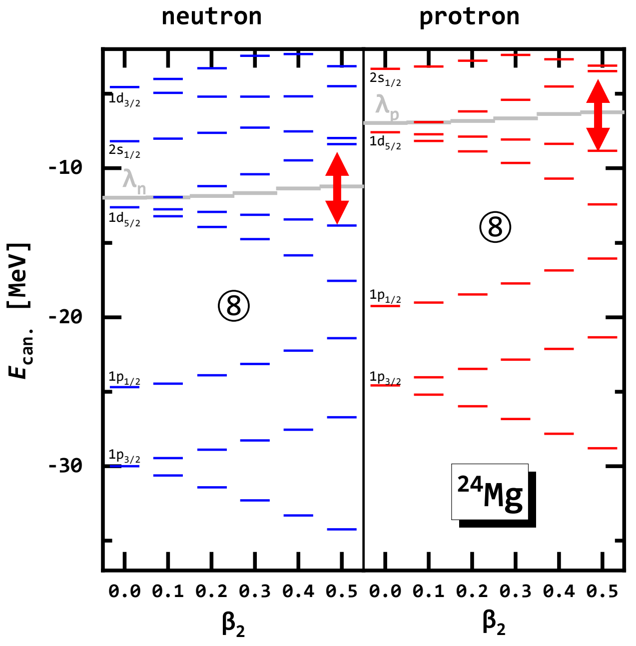
* numerical check are truncated by $N_{\text{max}} = 4$ for the calculation convenience.

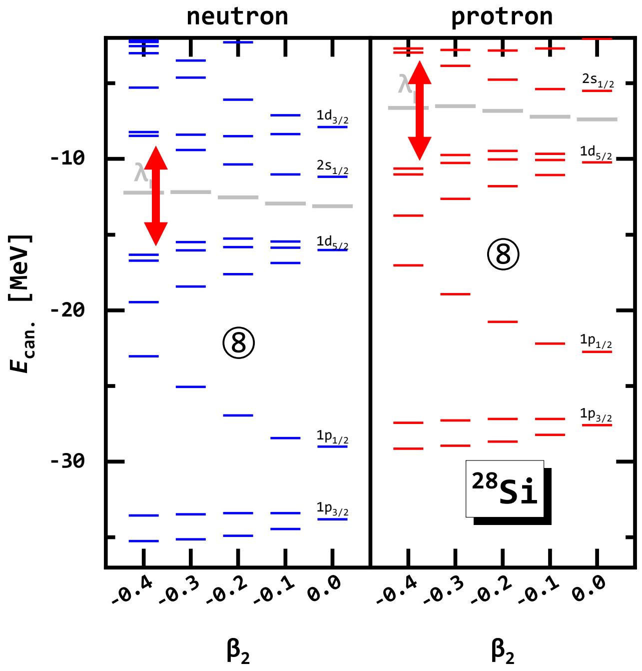
Preliminary results: Gamow-Teller Transition in ^{24}Mg & ^{28}Si

➢ Selected nuclides to illustrate the deformation effects:

	Theor.			Expt. from NNDC	
Nuclide	B. E. [MeV]	β_2	E_{pairing} [MeV]	β_2	Pairing Gap
^{24}Mg	−194.229	+0.526	0.000	0.606	4.600
^{28}Si	−232.233	−0.376	0.000	0.412	4.353

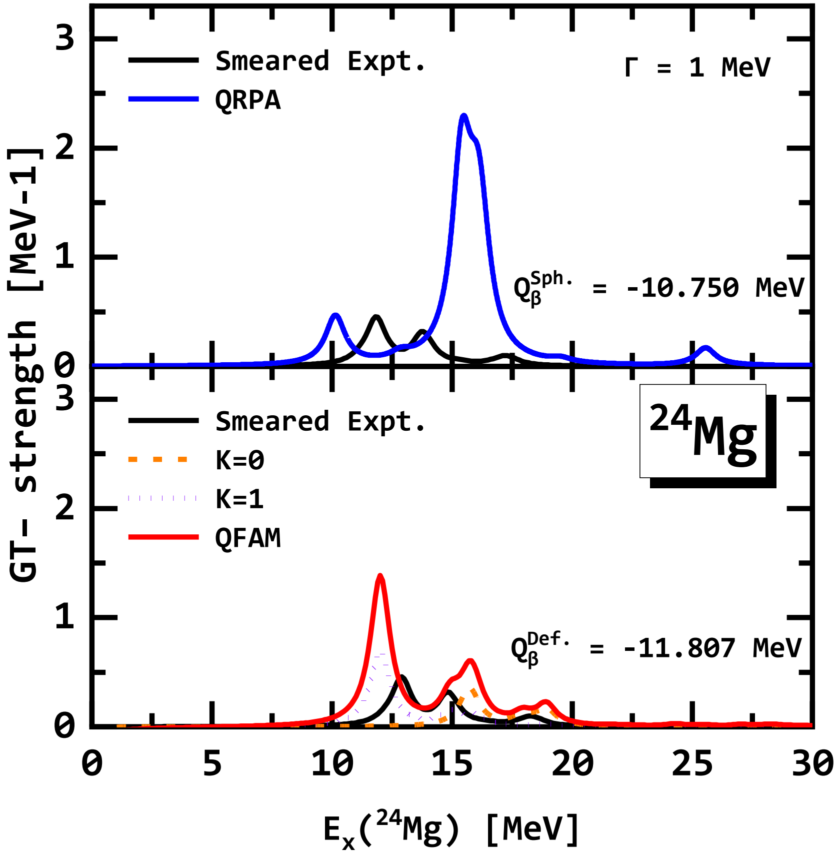
- Considerable gaps in the well-deformed region lead to negligible pairing effects.





Preliminary results: Gamow-Teller Transition in ^{24}Mg & ^{28}Si

➢ GT− strengths of ^{24}Mg



* ^{24}Mg(^{3}He,t) from R. G. T. Zegers et al. Phys. Rev. C 78, 014314 (2008).

** Expt. data shift by Q_β to compare with strength related to $E_x(^{24}Mg)$

*** $Q_\beta^{\mathrm{Def.}}$ given by deformed g.s., while $Q_\beta^{\mathrm{Sph.}}$ given by spherical g.s..

- GTR centroid energy: $E_{\mathrm{GTR}} = \frac{\int E_X\, S\, \mathrm{d}E_X}{\int S\, \mathrm{d}E_X}$;

Z. M. Niu. PhD thesis (2011).

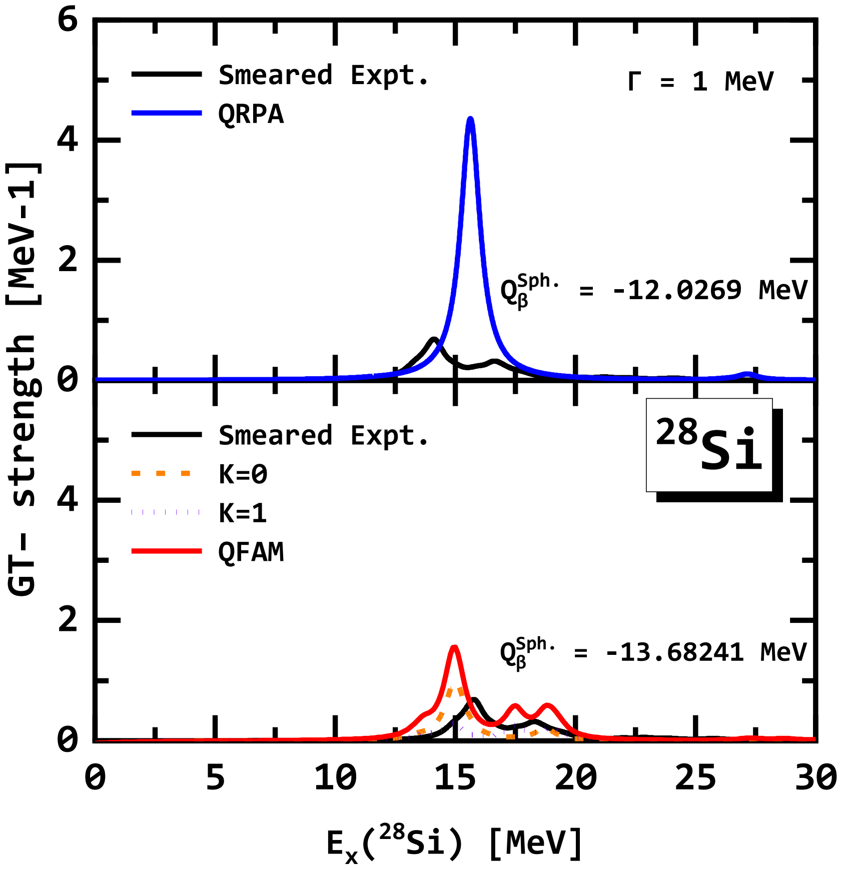
	$E_{\mathrm{GTR}}^{\mathrm{Expt.}}$ [MeV]	$E_{\mathrm{GTR}}^{\mathrm{Theor.}}$ [MeV]	$E_{\mathrm{GTR}}^{\mathrm{Theor.}} - E_{\mathrm{GTR}}^{\mathrm{Expt.}}$ [MeV]
Sph.	13.257	15.137	+1.880
Def.	14.275	14.103	−0.172

In spherical case, E_{GTR} is overestimated;
In deformed case, E_{GTR} becomes lower and closer to experimental data;

- The deformation leads to the splitting in GT− strengths which is closer to the experimental data, although the splitting is too much.

Preliminary results: Gamow-Teller Transition in ^{24}Mg & ^{28}Si

➢ GT− strengths of ^{28}Si



* ^{28}Si(p,n) from B. D. Anderson et al. Phys. Rev. C 43, 50 (1991).

** Expt. data shift by Q_β to compare with strength related to $E_x(^{24}Mg)$

*** $Q_\beta^{\text{Def.}}$ given by deformed g.s., while $Q_\beta^{\text{Sph.}}$ given by spherical g.s..

- GTR centroid energy: $E_{\text{GTR}} = \frac{\int E_X\, S\, \mathrm{d}E_X}{\int S\, \mathrm{d}E_X}$;

Z. M. Niu. PhD thesis (2011).

	$E_{\text{GTR}}^{\text{Expt.}}$ [MeV]	$E_{\text{GTR}}^{\text{Theor.}}$ [MeV]	$E_{\text{GTR}}^{\text{Theor.}} - E_{\text{GTR}}^{\text{Expt.}}$ [MeV]
Sph.	15.594	15.613	+0.017
Def.	17.011	16.129	−0.882

In spherical case, E_{GTR} is well reproduced, but without splitting;
In deformed case, E_{GTR} becomes lower and underestimated.

- The deformation leads to the splitting in GT− strengths which is closer to the experimental data, but the 1st peak has underestimated excitation energy.

Summary and Outlook

- Summary
  - We have preliminary developed a relativistic QFAM model for charge-exchange transitions.
  - Nuclear deformation leads to the remarkable splitting in GT strength;
  - Nuclear deformation results in a lower GTR centroid energy compared to the spherical case.

- Outlook
  - Completing the QFAM model by including pairing correlation;
  - Including the point-coupling functional with tensor coupling and with localized exchanged terms by Fierz transformation PCF-PK1;
  - ……

Thank you!

Appendix

- β_2-unconstrainted calculation: binding energies, deformations & peak energies