Ref: Phys. Rev. C 109, 065804
arXiv:2503.06250 [nucl-th]

Superfluid Band Calculations for Neutron Star Inner Crust

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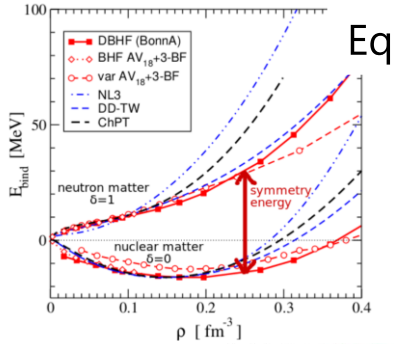
Neutron Star Physics

Theory

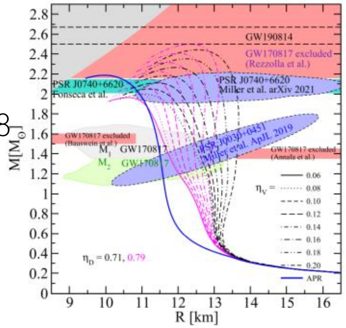
arXiv:2302.01453

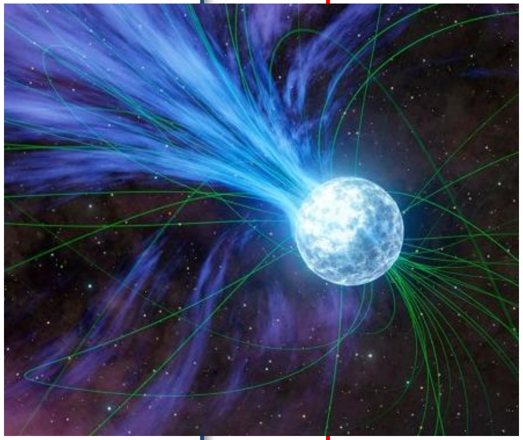
Equation of State



M-R relation

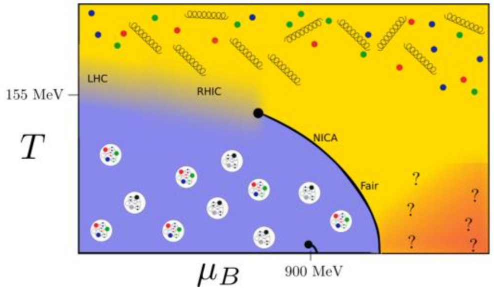
Eur. Phys. J. 57; 318





QCD Phase

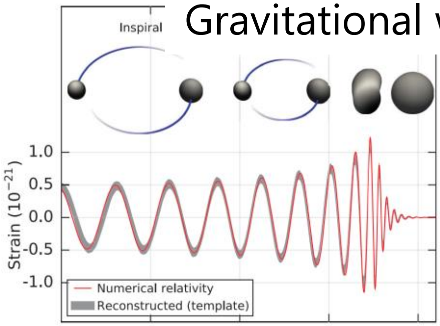
arXiv: 2010.15503



Observation

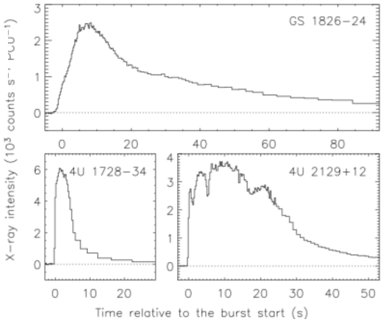
Phys. Rev. Lett. 116, 061102

Gravitational waves



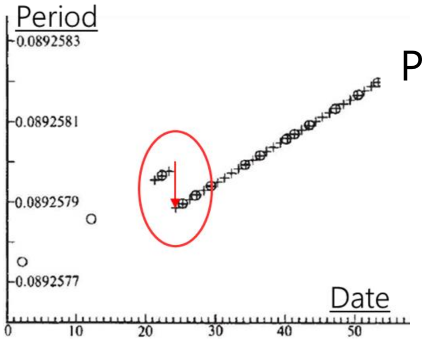
X-ray burst

Galloway+ 2008



Pulsar glitch

Nature 302, 292



N.S. kids in Sekizawa Lab.

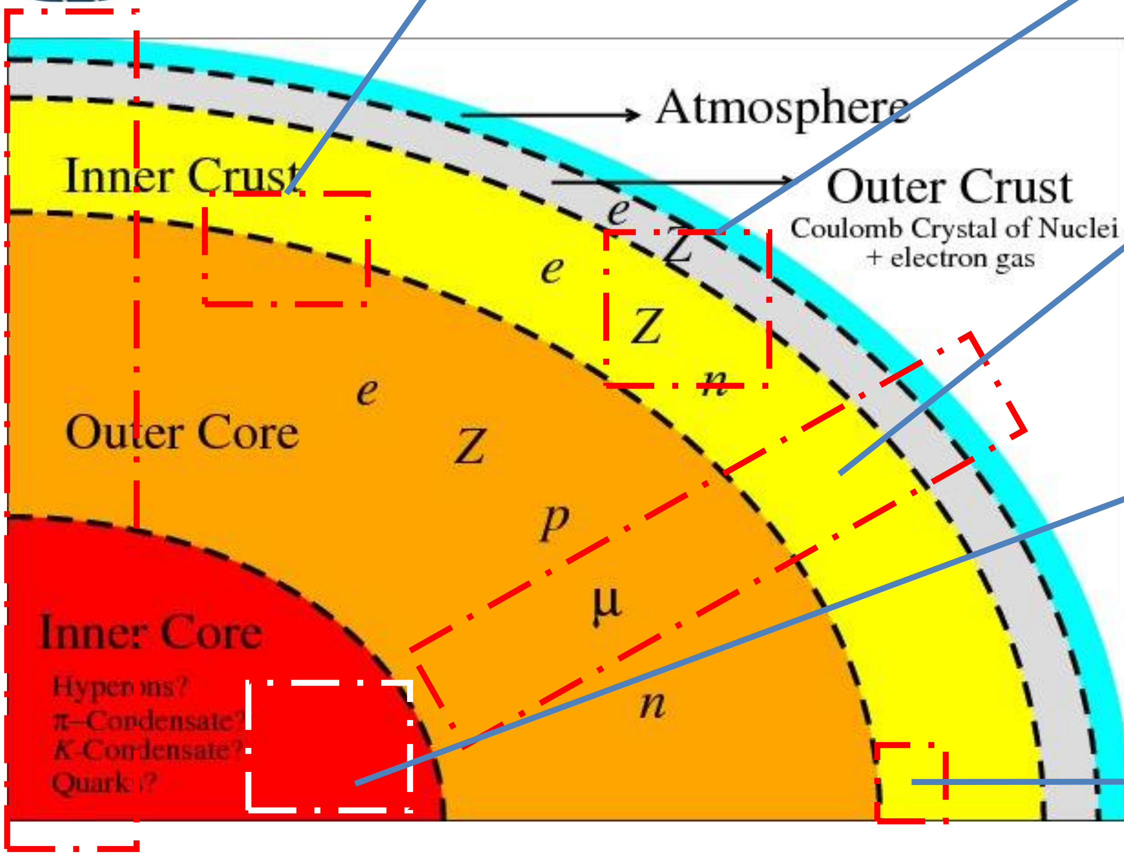
Kwon Hyukjin
rotating N.S.

Tatsuhiro Hattori
Dynamics of
Superfluid neutrons

Tateda Mahiro
Composition of
crust matter

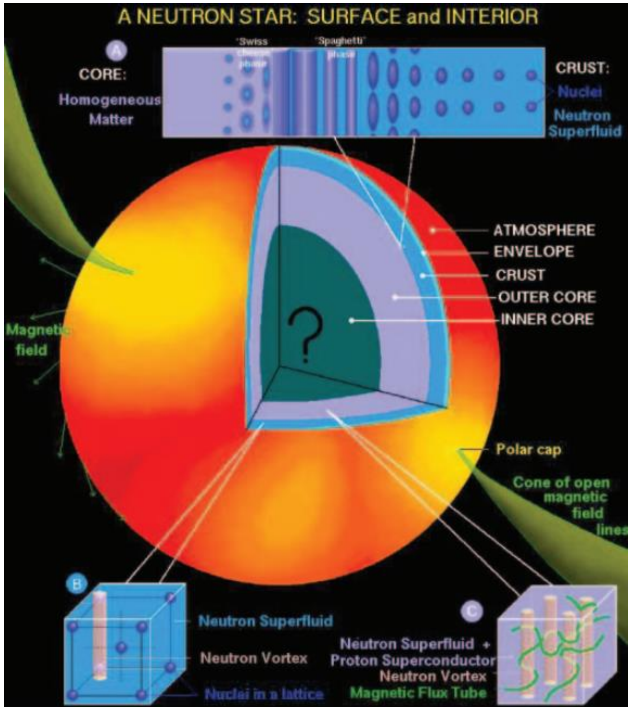
Nam Yoonhak
N.S. cooling

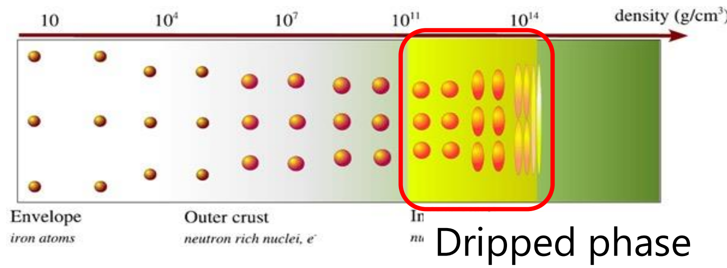
Lee Taeho
Hyperonic nuclear
matter

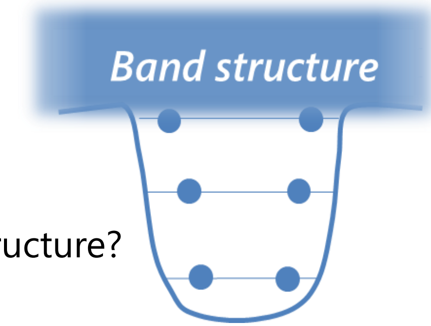


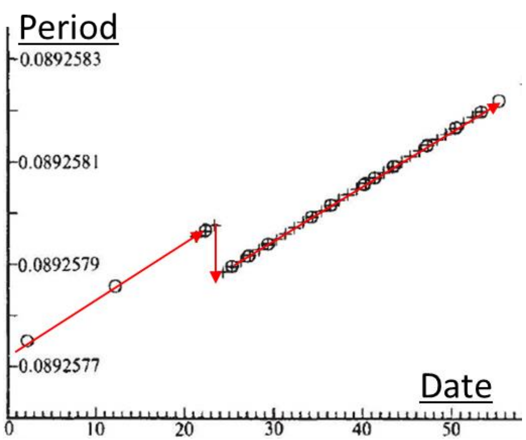
Me!

Neutron Star Physics









Nature, 302, 319 (1983)

Crust region

- Infinite systems of nuclei
- Source of astronomical phenomena
  - -- Pulsar glitches
  - -- quasi-periodic oscillations
  - -- Rapid nucleosynthesis
  - -- Supernovae
- Issues
  - -- Very neutron-rich
  - -- Neutron superfluidity
  - -- Band structure effects
  - -- Temperature, magnetar

Band calculations for inner crust

Band theory: quantum mechanics in a periodic potential

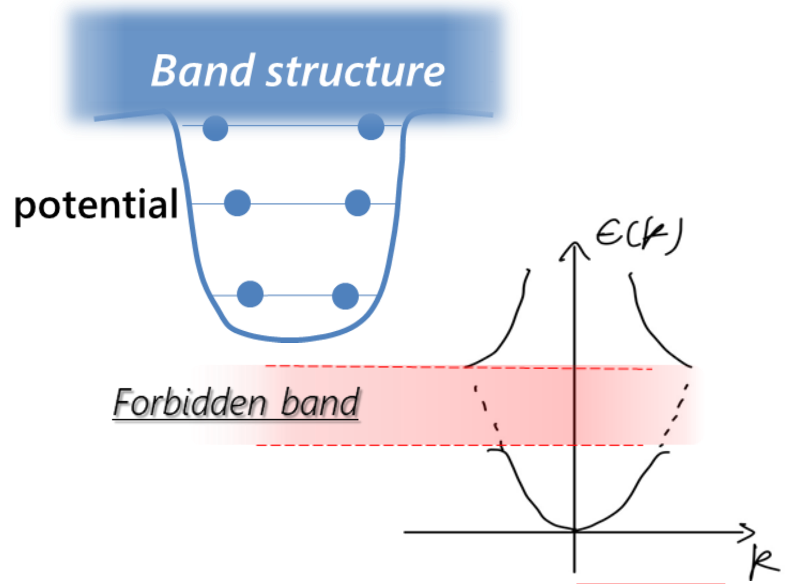
Question

What is the ***effective mass*** of free neutrons for nuclear pasta?

- N.Chamel, PRC85, 035801 (2012):

  Neutron effective mass is always larger than bare mass!

  (entrainment effect)



Entrainment :

- Dynamics of free neutrons terribly interfered

  ⇨ Neutron superfluidity reduced

$$n_s/n_n = m_n/m_n^\star$$

- Glitch hypothesis breaks down?

Issues :

- Not self-consistent with band theory
- Not consider pairing correlation (superfluidity)

$\bar{n}$ (fm^{-3})	Z	A	$n_n^{\rm f}/n_n$ (%)	$n_n^{\rm c}/n_n^{\rm f}$ (%)	$m_n^\star/m_n$
0.01	40	1215	88.9	15.5	6.45
0.02	40	1485	90.3	7.37	13.6
0.03	40	1590	91.4	7.33	13.6
0.04	40	1610	88.8	10.6	9.43
0.05	20	800	91.4	30.0	3.33
0.06	20	780	91.5	45.0	2.18

History of Band calculation

N.Chamel, PRC85, 035801 (2012)

effective mass calculations for pulsar glitch understandings

Self-consistency

Phys. Rev. C 100, 035804 (2019)

Phys. Rev. C 105, 045807 (2022)

- still w/o pairing
- only 1 dim. crystal
- effective mass ***less*** than bare mass

(anti-entrainment?)

Superfluidity

Phys. Rev. C 94, 065801 (2016)

Phys. Rev. Research 4, 033141 (2022)

- not self-consistent
- overestimation w/o self-consistency?

Our final goal :

1. develop ①self-consistent ②superfluid ③band theory
   for all crystalline structures realized in the neutron star inner crust
2. perform calculations and extract the ***effective mass*** of free neutrons
3. obtain the effective mass as a function of baryon densities,
   utilized for actual simulations for astronomical phenomena

Density Functional Theory

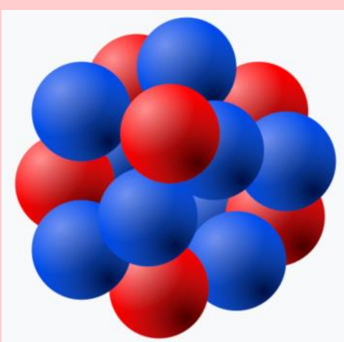
Nuclear Systems

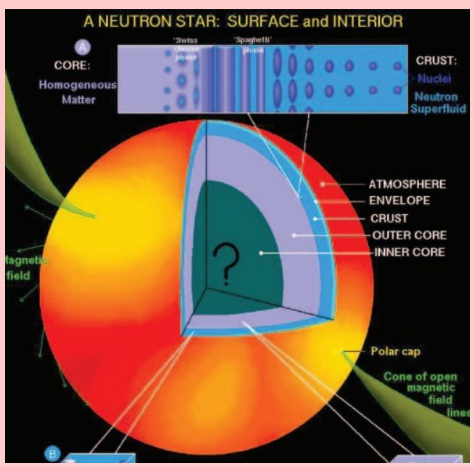
- Normally *finite* systems
- Skyrme-Hartree-Fock prevailing

$$\left[\frac{\hbar^2}{2M}\boldsymbol{\nabla}^2 + v^{\text{Skyrme}}(\boldsymbol{r})\right]\psi_i(\boldsymbol{r}) = \varepsilon_i\psi_i(\boldsymbol{r})$$

can be connected w/ DFT

- Band theory not integrated





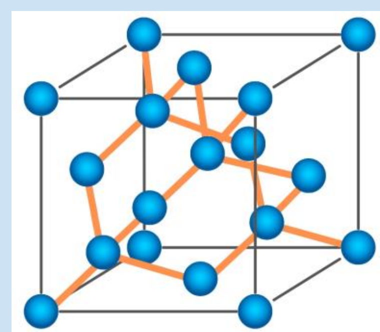
Electronic Systems

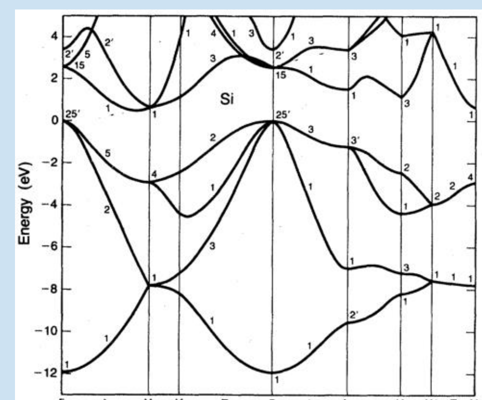
- Normally *infinite* systems
- KS-DFT successfully utilized

$$\left[\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + v^{\text{KS}}(\boldsymbol{r})\right]\psi_i(\boldsymbol{r}) = \varepsilon_i\psi_i(\boldsymbol{r})$$

by well-known Coulomb force

- For metals ***Band theory*** applied
- TDDFT for electron dynamics





Nuclear Band Theory

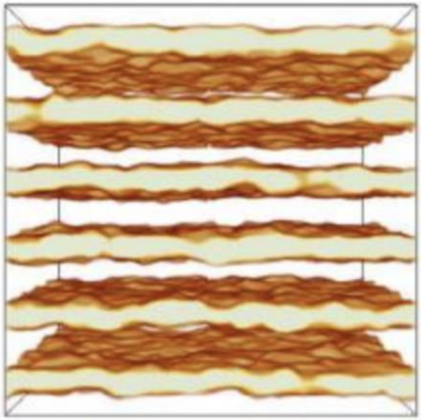
Nuclear HFB theory

$$\begin{pmatrix} \hat{h}_q - \lambda & \Delta \\ \Delta^* & -\hat{h}_q^* + \lambda \end{pmatrix} \begin{pmatrix} u_\mu^{(q)} \\ v_\mu^{(q)} \end{pmatrix} = E_\mu^{(q)} \begin{pmatrix} u_\mu^{(q)} \\ v_\mu^{(q)} \end{pmatrix}$$

Band Theory

$$\psi_{\mu\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \tilde{\psi}_{\mu\boldsymbol{k}}(\boldsymbol{r})$$

1D periodical phase



Band HFB equation

$$\begin{pmatrix} \hat{h}_q + \hat{h}_{q,\boldsymbol{k}} - \lambda & \Delta \\ \Delta^* & -\hat{h}_q^* - \hat{h}_{q,-\boldsymbol{k}}^* + \lambda \end{pmatrix} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix} = E_{\mu\boldsymbol{k}}^{(q)} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix}$$

There are so many orbitals in the computational space…

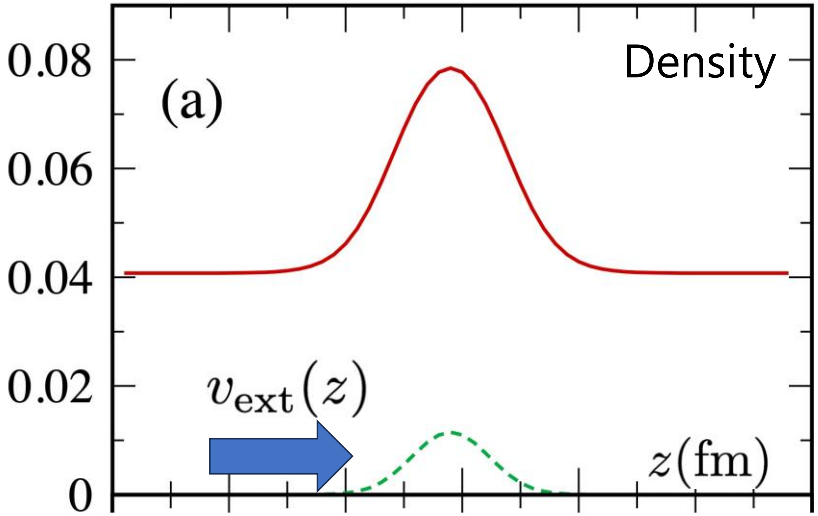
For 1D case, $N_z \times N_{k_\parallel} \times N_{k_z} \times (n,p) \times (u,v)$

$$\sim 60 \times 150 \times 80 \times 2 \times 2 = 2880000$$

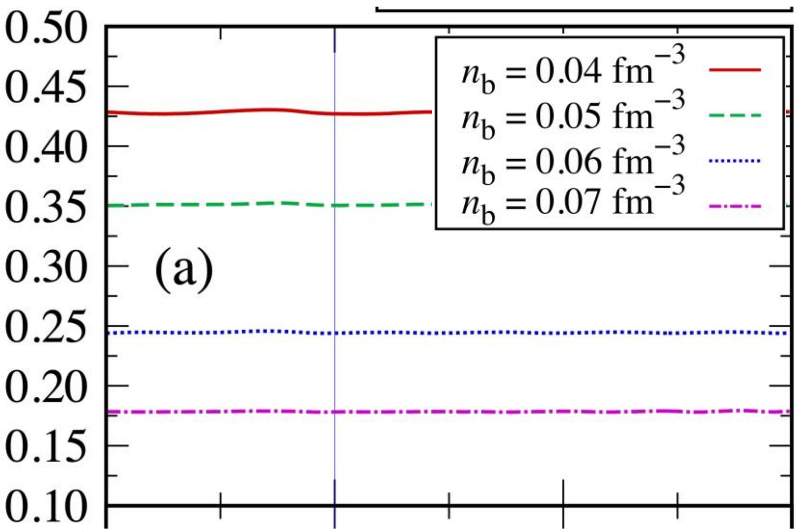
!!!????

→ ***SUPERCOMPUTING in CPU Parallelization !!!***

Anti-entrainment



Impose ext. field and calc. response…



Results less eff. mass than bare!!

	Superfluid (TD)DFT			Normal (TD)DFT		
$n_{\rm b}$	$n_n^{\rm f}/\bar{n}_n$	$n_n^{\rm c}/\bar{n}_n$	$m_n^\star/m_{n,{\rm bg}}^\oplus$	$n_n^{\rm f}/\bar{n}_n$	$n_n^{\rm c}/\bar{n}_n$	$m_n^\star/m_{n,{\rm bg}}^\oplus$
0.04	0.702	0.893	0.785	0.710	0.876	0.810
0.05	0.684	0.913	0.749	0.697	0.896	0.778
0.06	0.609	0.933	0.652	0.608	0.911	0.668
0.07	0.555	0.954	0.582	0.555	0.929	0.598

Takeaway!!

Band structure effects “enhance” the dynamics of neutrons, even with the superfluidity (*anti-entrainment* effect).

Formalism

HFB Equation

$$\begin{pmatrix} \hat{h}_q - \lambda & \Delta \\ \Delta^* & -\hat{h}_q^* + \lambda \end{pmatrix} \begin{pmatrix} u_\mu^{(q)} \\ v_\mu^{(q)} \end{pmatrix} = E_\mu^{(q)} \begin{pmatrix} u_\mu^{(q)} \\ v_\mu^{(q)} \end{pmatrix}$$

Band Theory

$$\psi_{\mu\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\tilde{\psi}_{\mu\boldsymbol{k}}(\boldsymbol{r})$$

Band-HFB Equation

$$\begin{pmatrix} \hat{h}_q + \hat{h}_{q,\boldsymbol{k}} - \lambda & \Delta \\ \Delta^* & -\hat{h}_q^* - \hat{h}_{q,-\boldsymbol{k}}^* + \lambda \end{pmatrix} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix} = E_{\mu\boldsymbol{k}}^{(q)} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix}$$

Densities (applicable to finite temp. systems)

$$\rho_q(\boldsymbol{r}) = \sum_{\mu\boldsymbol{k}} \left[f_D(-E_\mu)|v_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 + f_D(E_\mu)|u_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 \right] \quad \tau_q(\boldsymbol{r}) = \sum_{\mu\boldsymbol{k}} \left[f_D(-E_\mu)|\boldsymbol{\nabla} v_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 + f_D(E_\mu)|\boldsymbol{\nabla} u_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 \right]$$

Time-Dependent form

$$i\hbar\frac{\partial}{\partial t} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix} = \begin{pmatrix} \hat{h}_q(t) + \hat{h}_{q,\boldsymbol{k}}(t) & \Delta(t) \\ \Delta^*(t) & -\hat{h}_q^*(t) - \hat{h}_{q,-\boldsymbol{k}}^*(t) \end{pmatrix} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix}$$

Finite Magnetic-Field Extension

$$\hat{h}_q = \hat{h}_q^{(0)} + \hat{h}_q^{(B)} \qquad \hat{h}_q^{(B)} = -\left(\boldsymbol{l}\delta_{q,p} + g_q\frac{\boldsymbol{\sigma}}{2}\right)\cdot\tilde{\boldsymbol{B}}_q$$

w/ g-factors $g_n = -3.826$ $g_p = 5.585$

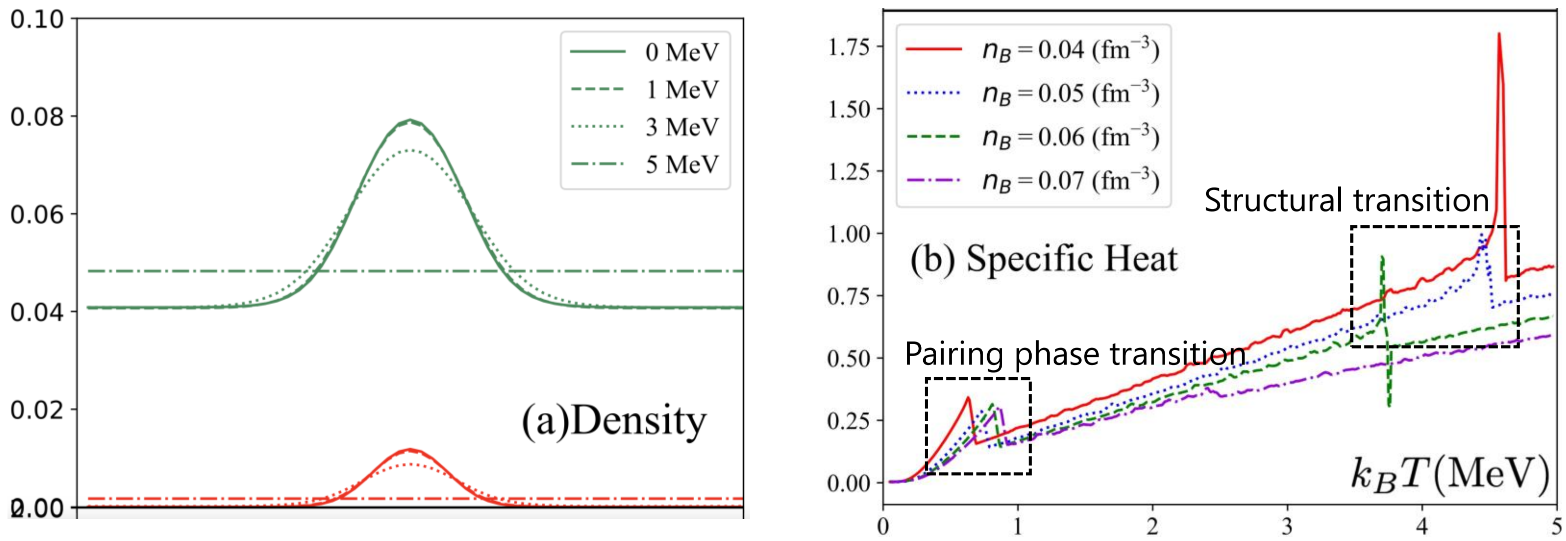
Finite-temperature, magnetic-field

Finite temperature form

$$\rho_q(\boldsymbol{r}) = \sum_{\mu\boldsymbol{k}} \left[f_D(-E_\mu)|v_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 + f_D(E_\mu)|u_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 \right]$$

Finite magnetic-field form

$\hat{h}_q = \hat{h}_q^{(0)} + \hat{h}_q^{(B)}$ $\quad \hat{h}_q^{(B)} = -\left(l\delta_{q,p} + g_q\frac{\boldsymbol{\sigma}}{2}\right)\cdot\tilde{\boldsymbol{B}}_q$ $\quad$ w/ g-factors $\quad g_n = -3.826 \quad g_p = 5.585$



Takeaway!!

Our framework is applicable to more realistic systems,
and can explore the phase structure of neutron star matter.

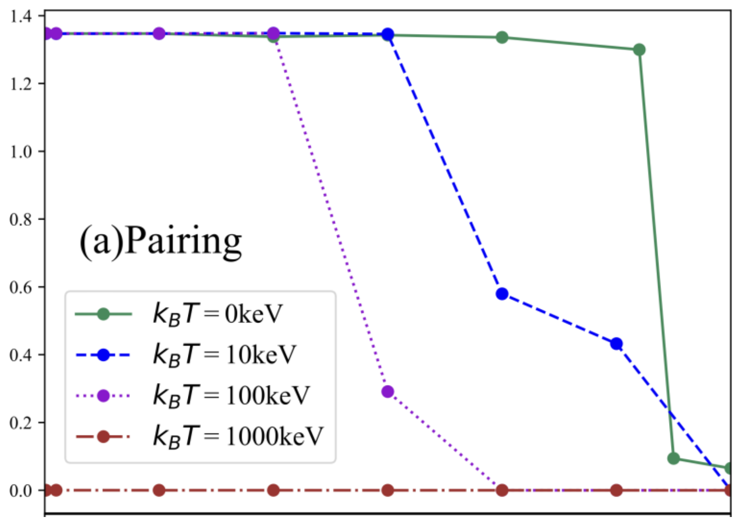
Finite-temperature, magnetic-field

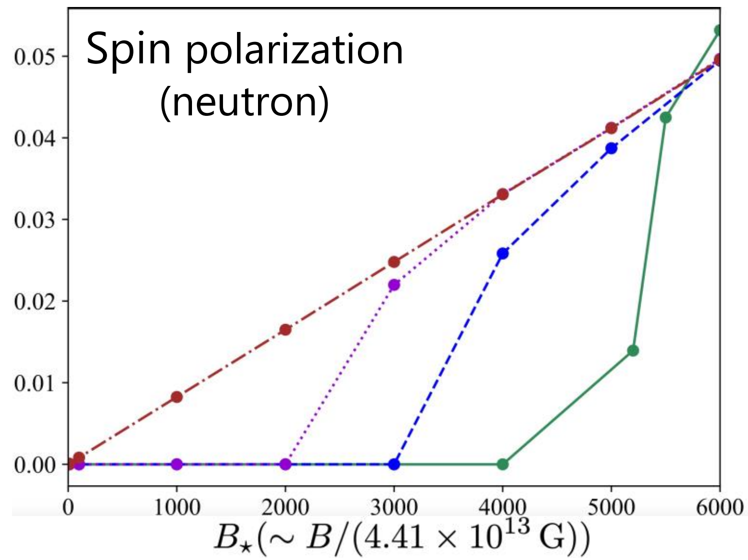
Finite temperature form

$$\rho_q(\boldsymbol{r}) = \sum_{\mu\boldsymbol{k}} \left[f_D(-E_\mu)|v_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 + f_D(E_\mu)|u_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 \right]$$

Finite magnetic-field form

$\hat{h}_q = \hat{h}_q^{(0)} + \hat{h}_q^{(B)}$ $\quad \hat{h}_q^{(B)} = -\left(l\delta_{q,p} + g_q \frac{\boldsymbol{\sigma}}{2} \right) \cdot \tilde{\boldsymbol{B}}_q$ w/ g-factors $g_n = -3.826$ $\quad g_p = 5.585$





Takeaway!!

Our framework is applicable to more realistic systems,

and can explore the phase structure of neutron star matter.

Towards further extensions

2D or 3D extensions are computationally challenging…

→ *More efficient comp. method necessary!!*

Normal HF calculation

Hamiltonian

$$\hat{H}^{\mathrm{KS}}\psi_i = \varepsilon_i\psi_i$$

Diagonalization

Re-input

eigenvectors

$$\{\varepsilon_i, \psi_i\}$$

densities

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i|^2$$

Shifted-COCG method

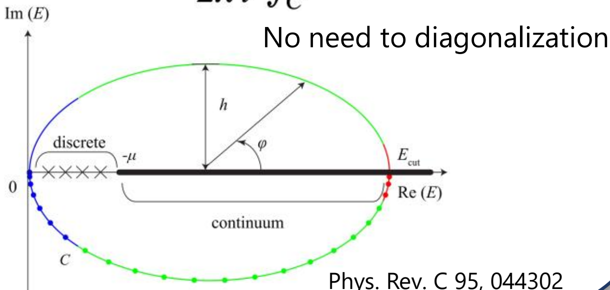
Green function

$$(z - H)G(z,\boldsymbol{r};\boldsymbol{r}') = \delta(\boldsymbol{r} - \boldsymbol{r}'),$$

Densities as contour integral

$$\rho(\boldsymbol{r}') = \frac{1}{2\pi i}\oint_C dz\, G(z,\boldsymbol{r};\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}$$

No need to diagonalization!



Phys. Rev. C 95, 044302

Takeaway!!

Our project on fully comprehensive band calculations

for neutron star inner crust is still being on-going!!

Summary and Prospect

What we've done

- Superfluid band calculations for neutron star matter within the inner crust
- Integrate nuclear HFB theory and band theory on the same footing
- Extend framework into finite-temperature and magnetic-field systems

What's been found

- Neutron dynamics is *enhanced* in the inner crust (*anti-entrainment*)
- With high temperatures two phase transitions take place
- With magnetic fields spin-polarized phase appears

What we plan

- Extend the framework into the 2D and 3D systems
- Complete the *table* of the Equation of State, and the neutron effective mass

Thank you for your careful attention